Boomerang Fractions

New Mexico

Supercomputing Challenge

Final Report

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Executive Summary

Boomerang fractions started as an interesting math problem proposed by at the Banff International Research Station Conference in 2015. It brings up the problem of fraction sequence's longevities. The term longevity represents the number of steps the fraction's sequence needs to return (boomerang) back to one.   First, you would add a chosen fraction, m/n/to one, and then either add that same fraction to the current number, or do the reciprocal of the current fraction. From there you would do another one of the two operations, creating another step, and you would continue to do so until the fraction reaches one. The fraction started at one and ended on one, thus acting like a boomerang. As previously said, the number of steps It took to get back to one would be its longevity.

The main goal of our project was to develop a computer program with the ability to realize the explicit longevities for individual fractions. The computer program was then used to help answer the following questions about the longevities of fractions: (1) What are the similarities in fractions that do not have longevities? (2) What are the contrasts between the fractions without longevities and fractions with longevities? (3) What are the similarities between fractions with greater longevities? and (4) What are the similarities in fractions with shorter longevities? We asked these questions in order to figure out why longevities work the way they do. We chose this project because when this problem was introduced to us, we developed these questions, and had an interest in how to solve them.

We chose to do our programming in Microsoft Excel since we knew how to program in Excel. Programming in Microsoft Excel allowed us to create a formula where we could then find solutions to our questions. We found that a fraction's longevity was created by the sequences needed to add to a whole number in order to do the reciprocal and have the correct denominator in which it could then add and reach one. This means that fractions without longevities are caused by the inability for the fraction to reach a state where it could then add and form a longevity; in that of needing to do the reciprocal first. This also shows that certain fractions with greater longevities are caused by the need for more steps to reach a state where it could then add and create the fraction's longevity. This works in the opposite sense; that fractions with a smaller longevity are caused by less steps required to reach a state where it could then add and form longevity, giving us the results needed to find the answer to our problem. We recommend in the future, the problem would be looked at more in depth, and to study more fractions, to more deeply investigate the differences and similarities in longevities.

Introduction

The basic premise of our project is that the sequences of fractions have a longevity. A fraction's longevity is the least amount of steps it takes for our sequence to resolve back to one. We start the sequence with the number one. After this the first step would be to add a chosen fraction to one. Next you would use one of two operations on the current fraction, one being to do the reciprocal of the current fraction, and the other would be to add the original fraction to the current one. The next series of steps would be to repeat step two in the sense of applying one of the operations, but to the fraction created in the previous step instead. You would do this until the fraction gets back to one. The number of steps it takes to get back to one is the fraction sequence's longevity.

step 1	step 2	step 3	step 4
3/2	2	1/2	1

(1) Add one half to one. (2) Add one half to one and one half. (3) Do the reciprocal of 2. (4) Add one half to one half.

The sequence is different for different fractions, also creating a large variation in longevities. One phenomenon we found was that certain fractions did not have a longevity, or they did not have one we could ascertain. There also seemed to be no commonality between why certain fractions have the longevity that they did. We found ourselves asking quite a few questions about the math problem. One example; What are the similarities of the fractions without longevities, another; what are the contrasts between fractions with longevities and fractions without longevities, third; what are the similarities in fractions with greater longevities, and finally what are the similarities between fractions with shorter longevities.

Our main objective in answering these questions was to understand why the fraction's sequence's longevities worked the way they do in their entirety. These questions would give us the knowledge and ability to eventually do so. They would create a basic layout of as to how boomerang fractions work, hopefully leading to a deeper understanding of the fraction's sequence's longevities in their entirety.

Code, Programming

We decided to take an approach to the problem by collecting a section of longevities. To do this we found that for our knowledge in programming Microsoft Excel would be the best program we could use, because we knew from previous knowledge that we could use a rand statement to help find possible solutions. This was important; because for our problem we needed a way to calculate multiple sequences for the same fraction in order to find the least amount of steps in a sequence, or its longevity. Microsoft Excel's application of cells also allowed us to differentiate steps, so that we could track the longevity. The rand statement would allow us to create multiple scenarios for possible longevities of each fraction. This gave us the ability to generate more possibilities of the sequence's longevity which then gave us a greater chance of finding the actual longevity, per calculation. We also knew that we had the ability to use an (IF) statement which gave us the ability to randomly generate the operations, which was crucial to our project. Without this we would not be able to randomly generate two operations, disabling us to form a sequence. Even though other languages like Net Logo have the ability to use these statements, for the purposes of our problem we found that it was unsuited for what we were doing. Though other languages may have benefited this project more, we do not know how to use them, thus making them irrelevant to our current situation, and making Microsoft Excel our best option. From this point we then started to try to create our basic code or formula for the project and came up with

=IF(rand()<0.5,[cell number]+[previous cell number],1/[cell number])

This gave us the ability to initially start one sequence, and from there our challenge was trying to find a way to create multiple trials on one sheet, and have the ability to change the chosen fraction without having to manually change the code each time. We did this by placing a cell with the ability to place a fraction in it, where it would then instantly copy that fraction into another cell where it would then add one to it, and from there copy that number as the first fraction in every sequence column, and from there we just applied the code listed above downward one hundred ninety six cells; giving it the ability to let a longevity reach one hundred ninety six steps, with two hundred six columns, creating two hundred and six scenarios for possible longevities. From there we programmed a cell to track all scenarios in order to list longevities, as well as a row dedicated to indicating what step that scenario reached one on. After we polished our program we then had the ability to find the longevity of a fraction's sequence after a few calculations. Thus, the program in Microsoft Excel allowed us to calculate the fraction's sequence's longevities.

Results

After creating our program in Microsoft Excel we then had the ability to find the fraction sequence's longevities at a more suitable rate for our problem. With our program we decided to find the longevity of fractions in between one half and eight ninths, finding a total of twenty-seven fraction's longevities. The fraction's sequence's longevities we found are shown in the table below.

1/2-4	1/3-9	1/4-7	1/5-20	1/6-6	1/7-49	1/8-13
1/9-25	2/3-5	2/5-13	2/7-24	2/9-13	3/4-6	3/5-28
3/7-17	3/8-11	4/5-7	4/7-NA	4/9-23	5/6-8	5/7-NA
5/8-15	5/9-NA	6/7-9	7/8-10	7/9-NA	8/9-11	

The fractions are shown with their corresponding sequence longevities in this table, showing the variety of longevities with each fraction. As previously stated, we found some fractions that behaved extremely well in the sense of having a lesser longevity, and some fractions going so far as to not have a longevity what so ever. After this we sought to find the answers to our objective questions, and started with similarities and differences in longevities in general. Some of the similarities, or patterns are listed below.

1. The fractions with the commonality N- 1 over N seemed to all have a lesser longevity, thus bring attention to themselves, so to compare them we decided to separate them onto another table in order to visually see the correlation and pattern between them.

1/2- 4	2/3- 5	3/4- 6	4/5- 7	5/6- 8	6/7- 9	7/8-10	8/9- 11	9/10-12	10/11-13

We instantly found that the fraction's longevities shared a linear increase, starting with one half. We continued to collect more longevities such as nine tenths and ten elevenths in order to see if the pattern continued, and we found that it continued to have a linear increase. This showed us that fractions with this commonality all shared a linear increase in longevity making the statement true, that when a fraction has the commonality of N-1 over N, the longevity is always (N-1) +3. As to why they functioned this way, we had yet to find out.

2. The next pattern we found in that of fraction's sequence's longevities was that there also seemed to be a commonality in fractions with the least possible numerator for each denominator (1). We found this pattern in looking in the opposite of the previous commonality, in that the fraction was the farthest away from the nominator, not the closest. Just as before, we created a table to visually display the commonality.

1/2- 4	1/3- 9	1/4- 7	1/5- 20	1/6- 6	1/7- 49	1/8- 13	1/9- 25	1/10- 16

In common with the other pattern every fraction had a longevity. These fractions did not have a linear increase, though we did find a pattern in the fraction's longevity. It seemed that all the fractions with this commonality had longevities which was holding up, though what wasn't was a clear pattern in the longevity. Eventually we found that the fractions with this commonality actually had a correlation with the denominator's factors. For instance, we saw that as two was a factor of four, one fourth had a similarly lower longevity, and that fractions that were prime without factors seemed to have a greater longevity, with the exception of fractions such as one half, one third, and one fifth, which had phenomenal longevities; making them seem to be the reason their multiples had great longevities as well, when fractions shared this commonality.

As we explored our data we were continuing to find answers to our objective questions, though they were not beginning to form an answer to our main question. Looking at the similarities and differences seemed to only allow us to observe the fraction's sequence's longevity's, not leading us to the answer of our main objective in this project. We realized we needed to look at another part of the data we had previously ignored, the individual steps of the longevity, which led us to make the following table including the sequence’s steps.

1 1/2	1 1/3	1 2/3	1 1/4	1 3/4	1 2/5	1 1/6	1 7/8
1 1/2	1 2/3	2 1/3	1 1/2	2 1/2	1 4/5	1 1/3	2 3/4
2	2	3	1 3/4	3 1/4	2 1/5	1 1/2	3 5/8
1/2	1/2	1/3	2	4	2 3/5	2/3	4 1/2
	5/6	1	1/2	1/4	3	5/6	5 3/8
1	1 1/6		3/4	1	3 2/5	1	6 1/4
	1 1/2		1		3 4/5		7 1/8
	2/3				4 1/5		8
	1				4 3/5		1/8
					5		1
					1/5		
					3/5		
					1		

This table brought to us an important part of our data, the reasons for our answers. This table led us to the discovery that all longevities always initially add, until reaching either a whole number or a fraction where it could then do the reciprocal in order to have either the same denominator as the first fraction, or a factor of the first fraction's denominator. From there it would then add again in order to reach one, because once the sequence did the reciprocal the current fraction now had the correct denominator or factor of denominator, and a numerator low enough to be able to add and reach one.

This showed us the reason that fractions with the first commonality had that relation; because it required one more step to reach a whole number to do the reciprocal, because the commonality required a numerator of one, which only a whole number could provide. This also showed us that fractions with the second commonality had the ability to do the reciprocal of a fraction other than a whole number allowing it to have a lesser longevity at times then it's first commonality counterpart, and it also showed us that because fractions with a prime denominator also needed to add until a whole number, and because the numerator is always one, it requires quite a greater amount of steps to add to a whole number where it could then do the reciprocal and add again to reach one. In finding this solution to the boomerang fraction's sequence's longevities we now understood the reason of as to why certain fractions had a lesser longevity (1). -

The fraction's numerator and denominator allows it to add to be able to do the reciprocal of a fraction in fewer steps, either by being able to use factorable denominators, or low enough denominators and high enough numerators, that the fraction could reach a state where it could do the reciprocal in fewer steps than other fractions. We could also understand why certain fractions had greater longevities (2). -The fraction's denominator is unable to reach a number where it could do the reciprocal in a lesser amount of steps because either the numerator is a factorable and/or the fraction's denominator is a higher than most, as well as the numerator possibly being lower in the fraction causing more steps needed to reach the needed number to do the reciprocal.

We also understood the answer to why certain fractions did not have a longevity (2). and (1). - The fraction's numerator in conjunction with the denominator causes the fraction to have the inability to add to a state where it could then do the reciprocal and have the correct denominator and numerator at the same time, disabling the fraction's ability to reach one; because it can never reach a state where It could then add and create the number one. This is due to the numerator and denominator causing the fraction to be unable to do the reciprocal at a factorable fraction because it either does not factor because of the denominator being prime, or the numerator being prime, though if one of these are not true then it cannot reach a longevity because the numerator is at odds with the denominator in the sense that it fallows none of the commonalities we found. We do not fully understand why these fractions do not have longevities. Overall, these fractions and their longevities are linked to the fraction's numerator and denominator, and their ability be factorable and have the ability to do the reciprocal at some point and have the both the correct numerator and denominator, to be able to add and reach one. The format of the fraction's denominator and numerator instantly depicts the probable length of its longevity, because if the denominator is a larger number it will most likely have a greater longevity from its predecessors, as well as if the fraction is factorable, though the numerator may change the longevity, if it follows one of the past commonalities, and if the denominator in conjunction with the numerator both have these negative characteristics, we have observed that it normally causes the fraction to have not have a longevity whatsoever. Thus, all fraction's sequence's longevities are affected by the fraction's ability to fallow the operations in the sequence and reach a step where it could then have the correct numerator as well as the correct denominator, and be able to then add the fraction and reach the number one.

Conclusion, Recommendations, and Acknowledgements

The problem of a fraction’s sequence’s longevities is formed around the number of steps required start at one, and then end at one. Finding the longevities by themselves using our program in Microsoft excel allowed us to observe patterns, and find the basic answers to our questions. However, we truly started to find the answers to our questions was in looking at the fraction’s sequences. We saw that the fraction’s sequence’s longevity was formed upon the number value of the denominator and numerator, which either allows the fraction to add, reach a state where it could correctly do the reciprocal, and add again to reach one, in fewer steps to reach longevity, more steps to reach longevity, or the inability for it to form a longevity. Over all this answered our question of as to why the longevities worked the way they did. Though, in the future we recommend that the longevity of more fractions be collected to possibly find more commonalities between the fractions, as well as to hopefully expand on the reason for fractions without longevities and why they do not have longevities, by finding commonalities in fractions without longevities and the key differences these fractions have, in comparison to fractions with longevities, and hopefully with this data, the clear reason these fraction’s sequence’s inexistence of a longevity exists. We hope in the future to explore these problems more in depth and to find the reasoning the number values effect the longevities so much and to hopefully explore fractions that are not reduced, to see if the longevity is maintained, an example being two fourths. All in all, this project has been beneficial to our overall understanding of fractions, sequences, and the correct exploration of projects as a whole.

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Bibliography

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