A Payload into Orbit

New Mexico High School
Supercomputing Challenge
Final Report
April 19, 2002

Team #058
Moriarty High School

Team Members:
Anna Reese
Odessa Gomez

Team Sponsor:
Paula Avery

Project Mentor:
Dr. RC Allen

Executive Summary

Since 1957 rockets have been used to carry payload into orbit. This project is a C++ program that finds the fuel needed to send a two-stage rocket into lower earth orbit (LEO) by varying payloads and exhaust velocities in the program. In order to do this the rocket theory equation is used: $v = c \ln R$ where the velocity, v, equals the exhaust velocity, c, times the natural logarithm of R, $\ln R$, (which is the weight to fuel ratio of the rocket). We are taking two standard payload weights and varying the engine efficiencies to test weights of fuel. We have successfully computed the results and have charted a graph that explains the changes in the payload with the changes in exhaust velocity. This program will be used to find the relationship between exhaust velocity and fuel. It will also find the relationship of the changes in payload to the changes in fuel. The program was designed to be applicable in the real world, with possibly government or commercial use. If we had more time we would also have incorporated other variables such as the efficiency of solid rocket propellant to liquid propellant and possibly the costs of the materials being varied.

Table of Contents

Introduction

Problem Definition

The purpose of our project is to determine if there is a relationship between payload, engine efficiency and the fuel weight of a rocket. This will help determine the costs of sending a two-stage lower earth orbit (LEO). This program also tests the engine efficiencies by varying the exhaust velocity of the engine and also how it affects the fuel weight and payload weight.

Background

The inspiration for this project came by the increase in technology and the growing need for more satellites as a result. Satellites play an important part in studies and measurements of the environmental phenomenon as well as gathering intelligence for agencies like the United States government and commercial corporations. National defense is also a growing concern with the events of September eleventh. When a space vehicle is launched into orbit, it needs to gain enough velocity to break away from the earth's gravity and atmosphere in order to get into lower earth orbit (LEO). The ability for rockets to escape the pull of gravity is called escape velocity. When an object is launched with an initial velocity greater or equal to the escape velocity, the object will go up, but it will not come back down because its velocity was greater than the pull of gravity. Thrust is the force that propels an object forward. The word propulsion is derived from the Latin words *pro* meaning before or forwards and *pellere* meaning to drive. Therefore propulsion means to drive or move an object forward. (Benson)

Propellants provide the thrust for the rockets. There are two kinds of propellant: solid and liquid. All propellants have a fuel and an oxidizer (oxygen). The most common fuel is either hydrogen or kerosene. The fuel and the oxidizer explode in the combustion chamber and produce a hot jet of gas that shoots out of the nozzle of the rocket. This provides the thrust for the rocket to move upward. The rate at which the exhaust comes out of the nozzle of the rocket is called the exhaust velocity.. The exhaust velocity can change depending on the size of the nozzle of the rocket. (Lobbia)

Principles

Rockets run on the principal of Isaac Newton's third law of motion that states "for every action there is an equal and opposite reaction." In this case the "equal and opposite" reactions are the exhaust velocity of the rocket engine and the pull of the earth's gravity. The minimum altitude for a stable orbit about Earth is about 160 km. At lower altitudes, air resistance slows the spacecraft and causes a rapid deterioration of the orbit. The spacecraft must attain a velocity of about 7.8 km per second to reach this altitude. However, in order to overcome the slowing effect of Earth's atmosphere the total velocity must be at least 9.0 (NASA Space Mathematics).

Assumptions

We have made the following assumptions in this project:

1. The structure weight of each stage is 10 percent of the fuel weight; the remaining weight being payload;
2. the gain in velocity is divided equally among the states, each contributing 4.5 km/s to the required final velocity of 9.0;
3. all stage us the same propellant with an exhaust velocity of 3.7 km/s; and,
4. the fuel will be burned to completion by the time the rocket reaches orbit.

Math Model

We used the basic rocket equation for this project:

$$v = c \ln R$$

Where:

v = velocity gained by the vehicle during launch

c = the exhaust velocity of the engine

$\ln R$ is logR or the natural logarithm of R

R = the mass ration of the spacecraft, defined by

R = takeoff weight/burnout weight or,

$$R = \frac{S + F + P}{S + P}$$

S represents the weight of the structure of the rocket

P represents the payload weight

F represents the fuel weight

w is used further in the equation to represent the weights of each stage

Each stage has its own set of variables:

Stage one is represented by S_1, P_1, F_1, v_1, w_1 and c_1

Stage two is represented by S_2, P_2, F_2, v_2, w_2 and c_2; in this case P_1 equals w_2 because the payload of stage one is all of stage two. The payload (P_2) in stage two is the actual payload the rocket is carrying.

We then used the above variables and put then in an equation to find fuel when given payload (P_2). We set S_1 to $0.1(F_1)$ and called the 0.1 *"ratio"*. The following are our equations and reasons.

$$P_2 + F_2 + S_2 = W_2$$
$$P_2 = W_2 - (F_2 + S_2)$$
$$= W_2 - F_2 - S_2$$

$=W_2 - (W_2 - SP_2) – ratio*F_2$ $SP_2 = S_2 + P_2$

$F_2 = W_2 – SP_2$

$=W_2 – W_2 + SP_2 – ratio*F_2$

$= SP_2 – ratio*(W_2 – SP_2)$ inserting W_2- SP_2 for F_2

$= SP_2 – ratio*W_2 + ratio*SP_2$ distribution

$= W_2/R_2 – ratio*W_2 + ratio*W_2/R_2$ When solving for R_2, it is found that $SP_2 = W_2/R_2$

$P_2 = (1/R_2 - ratio –ratio*R_2)*W_2/R_2$ distribution

****

$P_1 = (1/R_1 - ratio –ratio*R_1)*W_1/R_1$ The first stage is the same as the second

****

$W_2 = P_1$ Next we insert P_1 for W_2

****

- $P_2 = (1/R_2 - ratio –ratio*R_2)* (1/R_1 - ratio –ratio*R_1)*W_1/R_2*R_1$

****

$F = F_1 + F_2$

$= (W_1-SP_1) + (W_2 – SP_2)$

$= (W_1-W_1/R_1) + (W_2-W_2/R_2)$ substitution

$= (1-1/R_1)*W_1 + (1-1/R_2)*W_2$ distribution

$= (1-1/r_1)*W_1 + (1-1/R_2)*P_1$

from previous equation

$= (1-1/r_1)*W_1 + (1-1/R_2)* (1/R_1 - ratio –ratio*R_1)*W_1/R_1$

- $F= \{(1-1/r_1) + (1-1/R_2)* (1/R_1 - ratio –ratio*R_1)/R_1\}*W_1$

****

- Payload = $A*W_1$

when:

$A = (1/R_2 - ratio –ratio*R_2)* (1/R_1 - ratio –ratio*R_1) R_2*R_1$

****

- Fuel = $B*W_1$

when:

$B = \{(1-1/r_1) + (1-1/R_2)* (1/R_1 - ratio –ratio*R_1)/R_1\}$

****

W_1 = payload/A

****

fuel = $B*W_1$

- Fuel = B*payload/A

The above equations represent our program and the ability to find fuel from payload.

Computational Model

We wrote a C++ program that incorporated the rocket equation within loops. The first two loops varied the payload weight when given an exhaust velocity. We varied the payload weight between 1000 and 3000 kg. The two exhaust velocities we used were 3.7 and 4.1. The last two loops varied the exhaust velocity, when given the payload. We varied the exhaust velocities between 3.0 and 5.0**.** We chose to use 2600 and 2000 kg for the payload weights we tested. Each loop ran through a function, "fuelwt", which takes the given weight and velocity and runs through the equations to find, and return, fuel weight. Each loop then output the payload weight, exhaust velocity, and the resulting fuel weight for each run through.

Results

Our results show that the values for payload weights to fuel weights, at given exhaust velocities, are linear equations. Thus, we find that the greater the payload, the greater the needed fuel to put the payload in LEO. (See Appendix B)

We have found that the exhaust velocity also greatly affects the fuel weight. However, the exhaust velocity does not affect the fuel proportionally, as the payload does, rather, there is a curve. (See Appendix C)

Conclusions

Based on our results we have concluded that the engine efficiency of the rocket has a direct relationship with the amount of fuel needed to gain the required velocity of 9km/s. As the payload weight increases the fuel weight also increases. This also works the other way around, as the fuel weight increases the payload capacity will also increase. We have also concluded that there is also a relationship with the amount of payload weight that is available and the varying engine efficiencies with varying fuels. As the exhaust velocity increases the amount of needed fuel decreases. We have conclude, based on our results, that the ideal rocket exhaust velocity is between 3.5 and 4.5. This based is on the assumption that the best engine efficiency is expensive and difficult, so our conclusion observes the point where the curve rounds.

Future Work

There are many paths this project could take in the future. We might be able to take the take the relationship between engine efficiencies and the fuel and find the payload capacities of each. One might include comparing the efficiency of solid rocket propellants to liquid propellants, or add boosters. One could also find costs for the items and for better engines, and calculate the least expensive change with the most efficient engine. Another possibility would be to find the rocket structure (shape). The number of stages could be varied, and the optimal variation recorded.

Rocket science has many opportunities in today's world. We have but touched on the ideas of rocketry possibilities.

References

Kastner, Bernice, Dayle Seymour. Space Mathematics. CA; 1987.

Benson, Thomas. Beginner's Guide to Aeronautic. LTP. 3-19-02
http://www.grc.nasa.gov/www/k-12/airplane.

Benson, Thomas. Beginner's Guide to Propulsion. LTP. 3-19-02
http://www.grc.nasa.gov/www/k-12/airplane/bgp.html.

Benson, Thomas. Practical Rocketry. LTP. 3-19-02
http://www.lerc.nasa.gov/WWW/K-12/TRC/Rockets/practical_rocketry.html.

Lobbia, Robert. "How Rockets Work." Space Exploration November 23, 1997: Solid Propellant Rocket Engine. .
http://adc.gsfc.nasa.gov/adc/education/space_ex/rockwork.html

Lobbia, Robert. "How Rockets Work." Space Exploration November 23, 1997: Liquid Propellant Rocket Engine. .
http://adc.gsfc.nasa.gov/adc/education/space_ex/rockwork.html

Breaunig, Robert. Rocket and Space Technology. NASA. 10-31-01
http://users.commkey.net/Braeunig/space/.

, . How a solid propellant Rocket Works. NASA. 3-19-02
http://adc.gsfc.nasa.gov/adc/education/space_ex/solid.html.

**********Appendix A***********

Code

```
/*
  Name: Group #058
  Author: Anna Reese
       Odessa Gomez
  Description: In a 2-stage rocket, when given payload and the exaust velocity
          find the fuel necessary to reach Low Earth Orbit(LEO)
  */

#include <iostream.h>
#include <stdlib.h>
#include <iomanip.h>
#include <math.h>

float fuelwt( double payload_wt, double exhaust_vel){     // function which finds the
fuel weight when given
                                                          // payload and exaust velocity

//************declarations****************
float v1, v2, ratio, payload, a, b, c, d, e, f;          // variables needed for finding fuel
```

```
double x1, x2, r1, r2;                          // doubles in order to use them in
logrithems

//************* initialization ****************
v1 = 4.5;                  // change in rocket vel. stage 1.
v2 = 4.5;                  // change in rocket vel. stage 2.
exhaust_vel;                 // also called c-exhaust vel, stage 1 and stage 2
ratio = 0.1;                // ratio of structure being 10% of fuel weight
x1= v1/exhaust_vel;
x2=v2/exhaust_vel;
r1 = exp(x1);               // the mass ratio of first stage
r2 = exp(x2);
c = (1+ratio-ratio*r2)/r2;   // expressions used in payload expression for fuel
d = (1+ratio-ratio*r1)/r1;
e = (1-1/r1);
f = (1-1/r2);

a = c*d;                   // expressions used with payload for fuel
b = e+f*d;

double The_fuel_wt = (b/a)*payload_wt;   //declare and initialize fuel
  return The_fuel_wt;                  //return found fuel weight
}
```

```
//**********Start MAIN***********
int main()
{

//********DECLARATIONS********
 int fwt;                    // fuel weight inside main
 double exhaust_vel;           // exhaust velocity
 double pwt;                   // payload weight

 exhaust_vel=3.7;              // declare exhaust velocity at set 3.7 to be used in for loop
  // *********Begin For Loop***********
 for (pwt=1000; pwt<3010; pwt+=100){                        // *vary the payload from
1000 to 3000*
    fwt = fuelwt( pwt, exhaust_vel );                         // run the function to find fuel
weight
    cout << pwt << '\t' << exhaust_vel << '\t' << fwt << endl;    // print out
 }                                               // close loop


  exhaust_vel=4.1;             // declare exhaust velocity at set 4.1 to be used in for loop
// *********Begin For Loop***********
```

```
 for (pwt=1000; pwt<3010; pwt+=100){                              // *vary the payload from
1000 to 3010*
    fwt = fuelwt( pwt, exhaust_vel );                             // run the function to find fuel
weight
    cout << pwt << '\t' << exhaust_vel << '\t' << fwt << endl;    // print out
 }                                                            // close loop

 pwt=2600;                       // declare payload at set 2600 to be used in for loop
// *********Begin For Loop***********
 for (exhaust_vel = 3.0; exhaust_vel < 5.05; exhaust_vel += 0.1 ) {  // *vary the
exhaust velocity from 3.0 to 5.05*
     fwt = fuelwt( pwt, exhaust_vel );                            // run the function to find fuel
weight
    cout << pwt << '\t' << exhaust_vel << '\t' << fwt << endl;      // print out
 }                                                            // close loop

 pwt=2000;                       // declare payload at set 2000 to be used in for loop
// *********Begin For Loop***********
 for (exhaust_vel = 3.0; exhaust_vel < 5.05; exhaust_vel += 0.1 ) {  // *vary the
exhaust velocity from 3.0 to 5.05*
     fwt = fuelwt( pwt, exhaust_vel );                            // run the function to find fuel
weight
    cout << pwt << '\t' << exhaust_vel << '\t' << fwt << endl;      // print out
```

```
 }                                          // close loop



    system("PAUSE");                             // pause program to allow time to
see results
     return 0;                                 // return 0 from main
}                                          // end program and main
```

Acknowledgements

We would like to acknowledge all of the wonderful people who went out of their way to help us get the most out of our project. First we would like to thank our teacher, Mrs. Avery for all of the helping write our code and ideas for the report. We also are very grateful for the help we received from Dr. Richard A. Allen, in assisting with creating the formulas that we used in our program as well as the ideas for our program. We also would like to thank him for coming all the way out to Moriarty High School several times and giving us his time and patients.

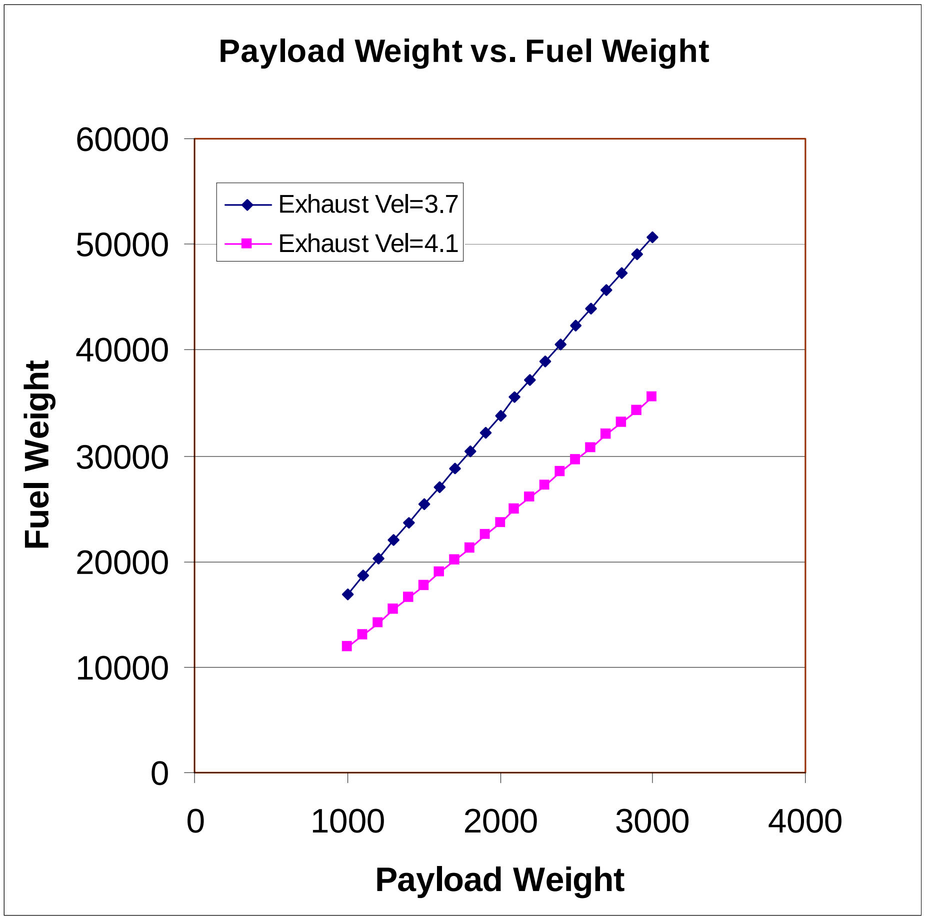
We'd also like to thank Rodger X. Lenard for his ideas and new information to add to our project. Also we would like to thank him for helping us to research and writing a model of two-stage earth-to-orbit paper that helped us understand more about rocket flight paths and about the NK33 Russian space rocket.

Also we would like to thank Jens Madsen II for coming in and helping our class with our presentations and also for personally helping us to take into consideration using the engine efficiency as a serious variable in our project.

We'd like to thank our principal, Dr. Walsh and school secretary, Lucille Milton for helping us out and for their support.

Appendix B

Graph of Results



Appendix C

Graph of Results