The Mechanics of a Volcanic Conduit from Inception to Eruption

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Executive Summary

Forecasting has never been an exact science, and volcanology is no exception. Like meteorology, medical diagnosing or the stock market, volcanology consists of many factors, both obscure and obvious, which add up to one major event. Attempting to describe a volcanic eruption precisely is nearly impossible, due to the variability that can occur with each case. However, like all the aforementioned fields, taking account of the obvious factors can usually lead to a fairly accurate of assumption of what is likely to occur. This project relies on taking the simplistic factors involved in an eruption and crafting them together to produce an approximated result, which although may not be substantially helpful in a real situation, can help the show the patterns and provide a more in depth understanding of the basic processes. Understanding is key in complex events, and any researcher can easily be bogged down when faced with so many diverse factors that affect a volcanic eruption.

This project analyzes the obvious factors which affect a volcanic eruption, under a set of assumptions that both limit the complexity of the problem as well as compensating for measurements that need at a specific field site. After reducing the processes into fairly simplistic forms, they are then mathematically modeled to show the processes which occur from a user determined beginning to the point of eruption. Using this model, an eruption can then be analyzed numerically and graphically, comparing the difference inherent in different situations. The model is also coded into an iterative solution,

allowing a user to see the correlation between time and the progress a potential explosive magma ascension.

Introduction

Eruption Basics and Background

Volcanic eruptions occur in two basic types, effusive and explosive. Effusive eruptions are typically very gas poor, meaning there are not many gasses initially dissolved or interacting with the ascending magma. The lack of gasses allows the Earth to very slowly and passively push magma towards the surface, and when it finally erupts it is more of an oozing out of the crater rather than what people normally associate with an eruption (some sort of explosive, deadly force. Effusive eruptions are the most studied, since they don’t come with sudden and deadly eruptions of magma, tremors or hazardous gasses. Frequent, ongoing effusive eruptions occur regularly in Hawaii, and can be safely observed by the average passerby. Explosive eruptions, on the other hand, are typified by gas rich elements, containing or interacting with volatile fluids (mostly water). These fluids dynamically tear the magma apart into pyroclasts (pumice and ash), and build up an enormous amount of pressure before literally exploding out of the ground. These eruptions typically have large shockwaves and tremors that occur with the eruption, and cause a significant amount of damage around the vent area. The eruptions of Mount St. Helens and Valles Grande in the Jemez Mountains are both well-known explosive eruptions.

This project is geared towards modeling the dynamic processes that lead up to an explosive eruption, since the level of complexity and lack of public information restricted generalizing the model further. Volcanoes have a long history in New Mexico, and even though most are now dormant much of New Mexico's landscape, geology, and environment were shaped by ancient volcanoes. This is most likely what caused the initial spark of interest in this project for this group, and influenced the decision in choosing it since it theoretically should have been a well researched topic in this area. Volcanology is, in fact, a well researched field, however most of the data and research is closely held to individual groups. The complexity of this field causes a fair amount of jealousy over data and accurate results, making specific data extremely difficult to access or even find. Most of the information for this project came from Ken Wohletz, a few obscure books found in the Los Alamos Laboratory's library and intuition.

Details & Limitations

Subsurface magma dynamics is a complex interrelation of physics and chemistry, including mass transfer between phases, processes such as nucleation, bubble growth and diverse events that theoretically trigger an eruption. Creating an accurate model of ascending magma requires calculating the various momentum changes between phases, heat and mass transfers, formation of gas bubbbles, etc. Combining all these makes modeling a very challenging discipline. Since time and information were both limited for this project, simplicity became the key issue in creating this model. To reduce the sheer amount of calculations, it was assumed that the magma ascent all occurred in a single phase, or rather all the phases occurred instantaneously at the moment of inception and cooperated as one homogenous body. The system is also assumed to be one dimensional as well as isothermal, and the geometry of the conduit is disregarded in interest of creating a generalized model. These combined assumptions create what is known as a steady homogenous model.

For explosive eruptions, the volatile fluid is assumed to be only water, and that it is initially dissolved in the magma (rather than interacting with it by means of an underground lake, groundwater, sea water etc.). Magma, lacking volatile fluids, can take years to reach the surface, and the exact force behind it is still debatably unknown. When a volatile fluid is added, what occurs within the magma is the formation of bubbles of gas which break apart the top section of the magma ascent. The theoretical solution is that these bubbles of gas are what provide the buoyancy within the magma and causes it to rise faster than in an effusive eruption. The solubility of the magma is mainly a function of pressure, and can be described using Henry’s Law. As the magma rises the pressure on the system decreases, causing the water to exsolve from the magma. This exsolved water is mostly what causes the eruption, and the pressure they exert on the system is what creates an explosive eruption.

The mechanics behind the exsolved water lie in understanding that as the water exsolves from the magma, it wants to vaporize and form steam (since magma is of course, very hot, with temperatures ranging from 900°C to 1200°C). Vaporization is also a function of pressure, and as stated by the Ideal Gas Law it requires pressure to contain a gas within a small space, and by Newton’s Third Law there is an also an opposite force exerted by the water onto the system. The main idea of the model created for this project is to describe how much the steam expands, and the force it exerts to push away the overburden above it. The speeds of this rushing column of gas, pyroclasts and magma can reach speeds faster than the speed of sound just before the eruption, creating an explosion very similar to that of a bomb. Volcanologists have long recognized that big explosive eruptions produce a mushroom cloud with a devastating lateral wind (called the base surge) that is very similar to that of a nuclear bomb, except with energies perhaps thousands of times greater than that of the Hiroshima bomb--the Mount St Helens eruption in 1980 had the energy equivalent to 7 to 20 Megatons, and the Jemez eruption would have been over a 1000 times greater!

Taking all this into consideration, the model initially assumes there is a constant riserate of the magma, with user determined depth. The pressure on the system is determined by a function involving the average density of the overburden, the depth and acceleration due to gravity. Plugging this pressure into Henry’s law gives the percentage of solubility, which gives the mass fraction of water per kg of magma. From here it is an iterative solution, involving how much the depth has changed and what the user has determined as the initial rise-rate. The model now simultaneously calculates the pressure and solubility at that depth, and the exsolved water is determined by how much the solubility has changed. From there the volume expansion is determined using water densities approximated from a set of Steam Tables (see Appendix B), and how much pressure the expanding steam is exerting is calculated using the Ideal Gas Law. Here, comparing the pressure exerted by the steam to the pressure exerted by the overburden added to the average rock strength, the model determines the status of the erupting volcano.

Results

An example of the output produced from the program is as follows:

```
Please enter the density of the rock (Kg/m^3):2500
Please enter the depth (m):10000
Please enter the solubulity constant:.13
Please enter the solubility exponent:.5
Please enter the rock strength (Pa):20260000
Please enter the density of the magma (Kg/m^3):2880
Please enter the temparature (K):1173
Please enter how high the magma has traveled(m):500
The percentage of dissolved water per kg of magma is 0.0623137%
The mass of exsolved water is 0.0016188 kg
Please enter the density of water under 2297.63 bars of pressure. 392.24
The pressure exerted by the steam is: .2193 bars
The total overburden is: 2497.63 bars
```

Showing that this particular volcano will erupt very shortly, with hardly any rise time.

The main idea behind this model shows how some magmas, depending on the constants that apply to them, how far under they are initially placed, etc. can react very differently. The above is based off a normal laboratory produced rhyolitic magma (used for small-scale testing purposes). Unfortunately the equations using to find the changing speed of the magma, or even the speed of the magma at the vent, were too finicky to use and kept producing unreasonable results. (The program keeps spitting out speeds on order of 20 million meters per second) The code is still intact, containing the faulty equation to show how some models just cannot be generalized. (This equation is taken off an experiment done in 1981 by L Wilson) However, when applied to the experiment that it was created for, it gave outstanding results, producing speeds around 400 meters per second, a speed fairly typical of a large explosive eruption. The experiment it was designed for had the eruption occuring very near to the surface, within around 800 meters and having around 40-60g of exsolved water (vs. the 1.6g that was produced in the example above.) Unfortunately since this is an approximated model it is difficult to determine whether or not it is an inherent fault in the math causing such a discrepancy between this model and the one produced in Wilson's model, because the results produced from the above example match similarly with what our Mentor gave us to work with.

Conclusions

Even when applied simply, the mechanics of a volcanic system are still extremely difficult to grasp, or model. One of the largest problems in dealing with information coming from this field is the fact that all the math is derived from a different basis, and it can be difficult to tell which equation uses what units. As far as can be discerned all the problems within the simpler part of the program were fixed to apply to the right unit, unfortunately the equations for determining the velocity or accelerations of the magma ascension are still dubious at best, since the authors lacked the foresight to include what units they were measuring their work with. The basics, however, are still modeled somewhat close to an actual volcanic system, even if it does ignore many of the major factors. Knowing the depth at which the magma will begin to break through the overburden above it is arguably the most important part of forecasting an eruption, since it is easier to monitor and record than the speed of ascension, bubble formation, etc. The surprising fact that the equation for the speed of the magma right before it erupts worked in a very limited situation was in itself quite a find for our group, since Wilson’s intent when deriving this equation was to attempt to make it generalizeable, and there is no mention that it does have limits in what he published. The other major success of this project was to bring a mass of obscure information together, and fit together with some sense of cohesion.

Despite the many disappointments and frustrations that occurred within this project, my group is fairly happy with the results we managed to produce. Forecasting the initial outbreak of a volcano is fairly easy, but almost no-one has been able to predict evolution, intensity, and some of the other goals we had in mind when starting this project. The most essential object of any prediction is whether or not the eruption will be effusive or explosive, something that can be determined by what we managed to produce. In analyzing the data and playing with the coefficients, it is easy to see that if the exsolved water is producing enough pressure to break through several kilometers of rock, it will be quite an explosion. However, if it is producing little to no pressure, and breaks near the surface, it is not as likely to be as dangerous and will most likely be a comparatively quiet, effusive eruption. Our mentor said we shouldn't try to get too far in over our heads in this field, and we have to say we should have listened to that advice a little more before trying out some of our various ideas. In his words, first order concerns are important and even necessary to a project like this, but when considering the second order, much more complex variables, it can be an extremely frustrating, time consuming wade through a mire of differential equations, mathematical abnormalities and such that the leading physicists in the world spend most of their lives working on.

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Appendix A:

Glossary of Terms

Volcano: An opening in the earth's crust from which hot magma reaches the surface

Conduit: The tube through which the magma rises

Explosive Eruption: An eruption characterized by gas rich magma

Effusive Eruption: An eruption characterized by gas poor magma

Homogenous: Everything inclusive; no separate parts

Magma Ascent: Formal name as magma is rising through the earth's crust

Pyroclasts: Ash and pumice issued forth from the eruption

Phases: The different stages of a magma ascent; Bubble-free flow, bubbly flow region, gas particulate flow region

Solubility: The percentage of dissolved water

Dissolved: The contained within the magma itself

Exsolved: The water that has separated from the magma

Appendix B: Data

Equations used:

$$P := \rho \cdot g \cdot h$$

Pressure

$$C := k \cdot P^{x}$$

Henry’s Law

$$M_{EG} := T_{G} - M_{DG}$$

Solution for mass of Exsolved water

$$V := \frac{M}{\rho}$$

Volume

$$P := \left(\frac{N}{V}\right) \cdot K_{B} \cdot T$$

Pressure exerted by Steam (Ideal Gas Law)

$$v := M_{EG} + \frac{\left[\left(1 - M_{EG}\right) \cdot P\right]^{\blacksquare}}{\rho_{lava} \cdot R \cdot T} \sqrt{\frac{(R \cdot T)}{M_{EG}}} \cdot \blacksquare$$

Velocity of magma at eruption point

P = Pressure
Δ = Density
g = acceleration due to gravity
h = height (or depth)
C = Solubility %
k = magma constant
x = magma constant (exponent)
M = mass
EG = Exsolved Gas
DG = Dissolved Gas
TG = Total Gas
V = volume
N = total number of molecules present
K_B= Boltzmann's constant
T = Absolute Temperature
v = velocity
R = Universal Gas constant

Appendix C: Program Code

```
#include <iostream.h>
#include <math.h>

class volcano
{
private:

	float prock;		// Pressure from overburden
	float psteam;		// Pressure exerted by exsolved steam
	float ob;	// Overburden of the rock (prock + pstr)
	float dwater;		// Density of Water
	float dmagma;		// Density of the Magma
	float volwater;		// Volume of exsolved water
	float drock;		// Density of rocks above magma chamber
	float temp;			// Temperature in Kelvin
	float rockstr;		// Strength of the rock
	float g;			// Acceleration due to Gravity (9.8 m/s^2)
	float d;				// Starting Depth of the system
	float solconst;			// Constant in Henry's law, n
	double solconstexp;		// exponent in Henry's law, x
	float solwater;		// amount of soluble water, c
	float m;				// Mass of the magma flow (in kg)
	float twater;			// total mass of dissolved water based off user input
	long float an;			// Avogadro's Number
	long float kb;			// Boltzmann's constant
	float molwater;		// Molar mass of water
	float v1;			// Velocity of system
	float R;			// Universal Gas constant
	float t;			// Time passed since inception
	float pbars;		// Pressure in bars (for Henry's law)
public:
	volcano();						// Constructor
	volcano( volcano& );			// reference constructor
	volcano( float, float);			// Constructor w/ calls for riserate and mass of flow
	void calcinitial();				// Calculates initial amount of water
	void calcsimple(float);			// Calculate simple solution (constant riserate), input
time
	void check();					// Check for eruption
	float timecalc();				// Calculate the time needed for an eruption

	friend istream & operator >>(istream&, volcano&);
	friend ostream & operator <<(ostream&, volcano&);
};
volcano::volcano()
{
	g = 9.8;
	an = 6.02e23;
	kb = 1.38e-23;
	molwater = 0.018;
	R = 8.31;
```

```
}

volcano::volcano(volcano & v)
{

        prock = v.prock;
        psteam = v.psteam;
        ob = v.ob;
        dwater = v.dwater;
        drock = v.drock;
        temp = v.temp;
        rockstr = v.rockstr;
        g = v.g;
        d = v.d;
        solconst = v.solconst;
        solconstexp = v.solconstexp;
        solwater = v.solwater;
        m = v.m;
        twater= v.twater;
        an = v.an;
        kb = v.kb;
        molwater = v.molwater;
        v1 = v.v1;
        dmagma = v.dmagma;
        t = v.t;
        pbars = v.pbars;
}

void volcano::calcinitial()
{
        prock = d*g*drock;
        pbars = prock/101300;
        solwater = solconst*pow(pbars, solconstexp);
        twater = m*(solwater/100);                                      // Total Initial water
}

//***********************************************************
//
// This next function could more logically be separated into
// more than one function, but for sake of time I have stuffed all the required
// calculations into just this one. The seperate steps are separated,
// solving for Pressure from the overburden, % solubility, mass fraction of exsolved
// water, volume change in steam expansion, Pressure from the steam, velocity at that
// depth, time to reach that depth, and finally comparison of whether or not it has erupted.
//
//***********************************************************

void volcano::calcsimple(float dd)              // dd = change in depth
{
        float cd;                                               // Current depth
        float cexs;                                                     // Current exsolved water
        float cdis;                                                     // Current mass of water
dissolved in magma
        float N;                                                // Number of particles of exsolved
water
        float x1, x2, x3, x4, x5, x6;           // Temporary placeholder to make equation simpler
```

```
    cd = d - dd;                            // Calculates depth of system after time t
    prock = cd*g*drock;
    pbars = prock/101300;
    solwater = solconst*pow(pbars, solconstexp);
    cdis = m*(solwater/100);
    cout << "The percentage of dissolved water per kg of magma is " << cdis << "%" << endl;
    cexs = twater - cdis;
    cout << "The mass of exsolved water is " << cexs << " kg" << endl;

    N = (cexs/molwater)*an;
    cout << "Please enter the density of water under " << pbars << " bars of pressure." << endl;
    cin >> dwater;                                        // This comes from Mathcad
(Linear regression problem)
    volwater = cexs/dwater;
    psteam = (N/volwater)*kb*temp;    // Calculates pressure of expanding steam
    x4 = psteam/101300;
    ob = pbars + rockstr;
    cout << "The total overburden is: " << ob << " bars." << endl;
    cout << "The pressure exerted by the steam is: " << x4 << " bars" << endl;

    x1 = (R*temp)/cexs;
    x2 = (cexs + (1-cexs))*prock;
    x3 = dmagma*R*temp;
    v1 = (x2/x3)*pow(x1,1/2);
    cout << "The velocity is " << v1 << " m/s." << endl;
    t = dd/v1;                                            // Instantaneous velocity, no
acceleration for us
    cout << "The time needed to reach this depth is " << t << " seconds." << endl;

    x6 = ob - x4;
    if( psteam != ob)
    {
        cout << "Your volcano has either already or erupted, "
                 "or is still rising towards the surface. The discrepency till eruption is:"
               << x6 << endl;
    }
    else
        cout << " Your volcano is ready to erupt." << endl;

}

void volcano::check()
{

    float r1 = psteam - ob;
    float r2 = ob - psteam;

    ob = rockstr + prock;

    if(psteam > ob)
    {
        cout << "The volcano has already erupted, invalidating these results." << endl;
        cout << r1 << " is the discrepancy above." << endl;
    }
    if(psteam = ob)
    {
```

```
                cout << "Right on the mark, you're volcano is ready to erupt." << endl;
        }
        else
        {
                cout << "The magma is still rising, give it some more time." << endl;
                cout << r2 << " is the discrepancy below." << endl;
        }
}


istream & operator >>(istream & is, volcano & v)
{
        cout << "Please enter the mass within the magma flow (in kg):";
        is >> v.m; cout << endl;
        cout << "Please enter the density of the rock (Kg/m^3):";
        is >> v.drock; cout << endl;
        cout << "Please enter the depth (m):";
        is >> v.d; cout << endl;
        cout << "Please enter the solubulity constant:";
        is >> v.solconst; cout << endl;
        cout << "Please enter the solubility exponent:";
        is >> v.solconstexp; cout << endl;
        cout << "Please enter the rock strength (Pa):";
        is >> v.rockstr; cout << endl;
        cout << "Please enter the density of the magma (Kg/m^3):";
        is >> v.dmagma; cout << endl;
        cout << "Please enter the temparature (K):";
        is >> v.temp; cout << endl;
        return is;
}

ostream & operator <<(ostream & os, volcano & v)
{
        os << v.prock << "  " << v.solwater << "  " << v.volwater << "  "
          << v.psteam << endl;
        return os;
}



void main()
{
        float d1;
        volcano v;
        cin >> v;
        v.calcinitial();
        cout << v;
        cout << "Please enter how high the magma has traveled(m):";
        cin >> d1;
        while(d1 > 0)
        {
                v.calcsimple(d1);
                cout << "Please enter how high the magma has traveled(m):";
                cin >> d1;
        }
}
```