The Perfect Engine

Adventures in Supercomputing Challenge

by

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E.0 Executive Summary

Small enhancements in thermodynamic efficiency have great influences in maximizing engine performance and operation. Because of a lack of the application of advanced mathematics with auto mechanics, many mechanics expend a great deal of time and money trying to generate superior engine performance and output. Most mechanics do not have the time or understanding to perform complex calculations that are necessary to evaluate the thermodynamic efficiency of an internal combustion engine. As a result, they undergo massive troubleshooting projects to attempt to increase engine efficiency. Many avoidable upgrades are expensively purchased when all that is actually necessary is an enhancement in current thermodynamic cylinder efficiency. The way to increase internal combustion engine performance is by increasing the thermodynamic efficiency of the engine.

This computer program enables users of the computer program to easily compute the thermodynamic efficiency of a four-stroke engine and provide a list of possible weaknesses that can be modified to increase the thermodynamic efficiency of the engine. The objectives of this project are:

- Generate a mathematical model of the Carnot and Otto Cycles
- Develop a computer program that calculates thermodynamic efficiency
- Identify the thermodynamic properties of the combustion process
- Evaluate change in efficiency with the use of parametric equations

These objectives are accomplished by creating a computer program that has a graphical user interface. By entering a few simple variables about engine specifications, the program output provides information about the engine efficiency and immediate solutions that increase engine performance. This is accomplished by comparing a model of the Otto cycle to a model of the Carnot cycle, using the same thermodynamic input parameters. Certain conditions, such as cylinder pressure and temperature, are compared to the typical conditions of the Carnot and Otto cycles so that the computer outputs speculations about possible weaknesses in the system.

The analytical results of the computer program are confirmed by hand-calculations. The mathematical models developed to describe the thermodynamic behavior of an internal combustion engine are employed to perform these calculations and the results are compared to the computer program output. In this manner, the computer program is verified ensuring that the program output is accurate and complete. The computer program is written in the C++ computer programming language. Calculations include interpolating equations of state from a chart of the Ideal Gas Properties of Air(1), inserting values into strings of equations, and comparing results and calculations. The results are then integrated into an Excel spreadsheet to enhance a real time presentation of the data. The visual representation in Excel consists of calculated graphs of the Carnot and Otto cycles.

1.0 Introduction

1.1 Purpose

The main purpose of this project is to develop a set of thermodynamic calculation models and a computer program, The Perfect Engine, using these calculations to evaluate the overall efficiency of a four-stroke internal combustion engine system. Two mathematical systems are applicable in order to investigate the maximum efficiency of the four-cycle internal combustion engine. These mathematically modeled representations of the internal combustion engine are the Carnot and Otto cycles. The Carnot cycle is a perfect demonstration of a four-cycle engine because it does not incorporate energy losses of any kind. It is therefore impossible to mechanically achieve an ideal efficiency that is demonstrated by the Carnot cycle. However, the Carnot cycle does provide a basis for comparison of other cycles, such as the Otto, that describe actual processes. Thermodynamic efficiency consists of a comparison of heat exchange between two bodies of temperature. When the difference between the two temperature bodies is high, efficiency becomes greater. In order to physically generate maximum efficiency as demonstrated by the Carnot cycle, as cold temperature approaches absolute zero, the greater the efficiency. It is not yet scientifically possible to reach absolute zero degrees Kelvin, so the Carnot efficiency of the four- stroke engine cycle cannot mechanically be achieved. However, the Otto cycle does entail physical imperfections in the four-stroke system such as heat, pressure and energy losses; therefore the Otto cycle is a more precise mathematical model and can be used to represent the actual thermodynamic conditions of a physical engine.

1.2 Scope

Initially, mathematical models of the Otto and Carnot cycles are created. The Carnot and Otto cycles (see Appendix 1 and Appendix 2), which are almost exactly alike, are systems of equations that compare the relationships between cylinder pressure, temperature, and internal energy in a four-cycle system. Because the Carnot cycle is simply the Otto cycle without included energy losses, a model of only the Otto cycle is created. By setting energy loss values to zero, the Otto cycle becomes a model of the Carnot cycle as well. The Otto cycle begins with a phase called adiabatic compression. Important specific values for this process, which are entered by the user, are:

- starting temperature
- compression ratio
- initial atmospheric pressure
- beginning internal energy

Once the piston inside the cylinder compresses the fluid to maximum compression, the second phase commences. This second phase is called the adiabatic expansion. Adiabatic expansion forces the piston back down the cylinder, almost back to the initial values of the system. Essential values for equations within adiabatic expansion are:

- maximum temperature

- pressure
- internal energy

The third process is known as isobaric exhaustion. The heated fumes and burnt fuel of the system is discharged from the cylinder as the momentum of the engine forces the piston back up to maximum compression once again. The cylinder is almost completely emptied of gases during this phase. Necessary values for equations within isobaric exhaustion are:

- temperature
- pressure

The fourth and final sequence is known as isobaric intake. As the momentum of the engine pulls the piston back down the chamber, an intake valve opens, allowing a new gas/air mixture to fill the cylinder. Because the spark plug does not fire and gases should not be compressing during the isobaric exhaust and intake processes, the cylinder should be experiencing cooling. The third and fourth processes combined set the conditions relatively close to the initial conditions of the cycle. Values needed in equations within isobaric intake are:

- temperature
- pressure

The entire Otto thermodynamic system is an evaluation of the exchange of heat from hot to cold. Because the minimum temperatures are relatively close to each other, and the maximum temperatures are relatively close to each other, the cycle is known as thermodynamically reversible. The cycle recommences after every four phases.

By mathematically modeling the Otto cycle, the thermodynamic efficiency is calculated. The Perfect Engine computer program enhances understanding of the thermodynamics of the total Otto and Carnot cycles and thereby identifies measures that can be taken to improve the overall efficiency of an internal four-stroke internal combustion engine.

1.3 Computer Program

The computer program is written using the C++ programming language. This language can perform large calculations and store a large inventory of variables in arrays. Loops are created in order to perform various calculations involving computational data collection. Interpolation from a chart (see Appendix 5) is required to gain values for essential equations of state, which are assumed to be correct throughout the entire system. This means that all situations are expected to react normally under specific conditions. Users of the computer program are able to select data values from the chart rather than manually solving equations of state. Graphs, equations, and charts of the Otto and Carnot cycle are studied to understand and create a model of thermodynamics. All C++ computations are checked by hand calculations for the purpose of verifying the accuracy of outputted results and conclusions. Along with a working model of thermodynamic efficiency, real time visual representations in the form of graphs and charts are outputted using the Excel programming language.

2.0 Problem Statement

2.1 Problem Statement

Efficiency of a four-stroke internal combustion engine is nearly completely measured by thermodynamics. Obstacles in computing thermodynamic efficiency and yielding accurate inferences for engine thermodynamic efficiency magnifications are:

- Identification of required input variables
- How to selectively input the parameters
- Understanding outputs
- Creating visual representations of the models

The Perfect Engine computer program is designed to overcome those restrictions so that mechanics are able to acquire rapid and precise recommendations for improving four-cycle engine thermodynamic efficiency.

Providing a technique for enhancing knowledge of the internal combustion engine efficiency and a process for quickly evaluation the overall thermodynamic efficiency is the goal of this project. In order to achieve this, a comprehensive set of equations are developed that describe the complete thermodynamic cycle of an internal combustion engine (assumed to be operating on the Otto Cycle) which then requires that selective variables be identified that impact engine efficiency. Parametric equation operations are perforemed on each individual variable to determine the overall effect or impact each variable has on the comprehensive equation.

3.0 Method of Solution

3.1 Mathematical Model

The developed C++ computer program (see Appendix 3) is a working mathematical model of the thermodynamic equations that descrive the four internally reversible processes of the Otto cycle. These phases or states are commonly called compression, power, intake and exhaust.

The comprehensive equation that mathematically models the Carnot cycle is:
- $\eta = 1 - (T_{low} / T_{hi})$

The comprehensive equation that mathematically models the Otto cycle is:
- $\eta = \{ [T_{low} * \ln (V_d / V_c)] / [T_{hi} * \ln (V_b / V_a)] \} + 1$
- $\eta = (Q_{hi} + Q_{low}) / Q_{hi}$

where (for both equations):
- η = efficiency
- T = temperature (maximum and minimum for each cycle – in degrees Rankine)
- V = volume (in cubic feet – also is basis of compression ratio)
- Q = work (input and output)

The Excel portion of the computer program (see Appendix 5) displays a real-time graph using the calculated values that represent each phase of temperature and compression. With this working display of a four-stroke engine cycle, mechanics observe how efficient an engine really is. Hand calculations are performed to ensure accurate results and conclusions.

3.2 Computational Methods

With this computer program, an enhanced knowledge of internal combustion engine efficiency is quickly evaluated and easily accessed. This program allows users to input a minimum of required variables to receive approximations for quick readings of engine efficiency. Also outputted by the computer are approaches for broadening engine efficiency. In order to accomplish this aspect of analysis, it is required that the computer program assumes that each distinct cylinder phase obey, to an extent, the typical gas equations of state. This means that the computer program does not incorporate particular situations where certain engine conditions alter the normal physical properties of the system. Therefore, as calculations are performed and results are given, the information outputs are required to be valid enough to permit accurate conclusions, unless engine status has missing or additional components. Therefore, The Perfect Engine computer program allows mechanics to determine how close an engine is to reaching peak efficiency; and furthermore, with this computer program, mechanics can obtain modern guided conceptual reasoning for increasing thermodynamic efficiency by innovating new engine prototypes that control conditions and environments more effectively.

The Perfect Engine computer program computes the four specific phases involved in a four-stroke engine (see Appendix 3 and Appendix 4).

The computer program requires only four simple variable inputs:

- Initial Temperature (Tb) - lowest gas temperature in the cylinder
- Compression Ratio (r) - area of the volume compared to compression distance
- Compression Temperature (Th) -hottest temperature of the system
- Initial Atmospheric Pressure (p1) – starting atmospheric cylinder conditions

Each calculation in the program requires answers from the previous calculation to produce the next. Variables required for these calculations are:

- V2 - volume isentropic compression
- V3 - volume at hottest temp
- V4 - volume at isentropic expansion
- max - exact hottest temp
- min - exact lowest temp
- Ta – temperature at isentropic compression
- T4 - temperature at isentropic expansion
- minV - lowest volume of intervals
- maxV - highest volume of intervals
- p2 - pressure of atm at isentropic expansion
- p3 - pressure of atm at power stroke
- p4 - pressure of atm at isentropic expansion
- U1 - internal energy at lowest temp
- U2 - internal energy at isentropic compression
- U3 - internal energy at hottest temperature
- U4 - internal energy at isentropic expansion
- TE - Thermal Efficiency

All calculations from the computer program result in the output of the thermal efficiency of the specified engine conditions. Also produced is the engine's comparison to the thermodynamically perfect Carnot cycle, and graphs to visually demonstrate weaknesses and imperfections. Finally, the program outputs possible solutions and probable causes of deficient engine performance.

Equations of state are necessary to commence basic equation calculation and efficiency evaluation. Values for these equations of state are computationally interpolated from a chart. The computer program reads numbers from the chart and calculates similar values among other variables. The values are inserted into a system of loops, arrays and calculations that result with thermodynamic efficiency for specific engine conditions. Fundamentally, the interpolation from the chart evaluates how the cylinder conditions should react under certain environmental factors. Then, calculations are performed to output thermodynamic efficiency.

4.0 Results

The computer program calculates thermodynamic engine efficiency, the temperature at isentropic expansion, temperature after isentropic compression, the first pressure of the ideal gas state, and the second pressure of the ideal gas state. Once these values are accounted for and compared to ideal circumstances, speculations involving cause and effect are assumed. Parametric equations are constructed to infer conclusions about certain case conditions. By changing one variable at a time, observations to the different output values can be made.

4.1 Calculations

All computer calculations are proven correct by comparing computer results to hand written calculations. A significant difficulty encountered in the initial development of the program was the extraction of data from the chart of ideal gas properties of air (see Appendix 6) and presenting in a temperature dependent format for input to the appropriate computer equations. The computer program uses these complex interpolation values in order to begin the sequential process of computing the thermodynamic efficiency.

As those multiple values are calculated by interpolation, series of equations unfold to finalize by outputting the current thermodynamic efficiency of the engine. As each series of equations finds values important to the next set of equations, those values are outputted so the user understands how the detailed thermodynamics of the engine are functioning.

Computer calculation results follow this format:

- user is prompted to input:
    - initial cylinder temperature
    - compression ratio
    - melting temperature of the engine
- computer interpolates from chart:
    - temperature
    - volume
    - specific internal energy
    - pressure
- interpolation values are inserted into these previously shown sequential equations:
    - $\eta = 1 - (T_{low} / T_{hi})$
    - $\eta = \{ [T_{low} * \ln (V_d / V_c)] / [T_{hi} * \ln (V_b / V_a)] \} + 1$

4.2 Graphs and Figures

Graphs of the Carnot and Otto cycles are shown in Appendix 1 and Appendix 2. Both represent a four-stroke engine. The Carnot cycle is a thermodynamically perfect model of a four-cycle engine. It is a representation of the greatest efficiency that can be obtained for a given set of variables. This is because the Carnot cycle does not take into consideration energy losses due to pressure, heat, friction and other mechanical imperfections. Consequently, to mechanically reach this level of efficiency is impossible, but measuring the efficiency of a four-cycle engine is an excellent way to demonstrate how advanced a specific engine system is in the modern world.

The Otto cycle is used to create a working model representation of a four-stroke internal combustion engine. The Otto cycle does take into consideration energy losses, and is therefore a thermodynamically imperfect yet physical system. Both the Otto and Carnot cycles are descriptive relationships between cylinder pressure, temperature and internal energy. The Otto cycle covers all four strokes, which are intake, compression, power, and exhaust. Graphs, which are outputted by the computer for specific value inputs, demonstrate the beginning of each phase with four labeled points. (see Appendix 4)

5.0 Conclusions

5.1 Mathematical Models

Through the process of interpolation, calculation and graphical representation, thermodynamic efficiency is accurately presented. All calculations and computer outputs are proven correct by written hand calculations.

5.2 Computer Program

Calculating the thermodynamic efficiency of an engine requires selected basic inputs, value parameters known as equations of state, and a series of specific equations. The computer program initially prompts the user to input four easily referenced variables from either a *Chilton's Manual*(2) or a *Specifications Log*(3). Using these basic variables to specify the unique characterization of the engine, a programming loop interpolates values from the previously mentioned chart of values in order to provide the equations with accurate information on how the gases inside the cylinder react under certain conditions. Again, these are known as equations of state. With the equations of state and the variable inputs from the user, calculations in the form of loops are performed to examine the thermodynamic efficiency of the engine for a variety of modifications to the starting conditions of the thermodynamic cycle.

5.3 Results

Comparisons with the Carnot cycle tell the user how efficient the engine is compared to a perfect system. This is done in the form of graphs and numerical outputs (see Appendix 4). Another very useful feature of the computer program is the program's ability to provide the user with estimations as to what kind of engine problems might be causing the engine to lack in performance. As the computer program calculates the thermodynamic efficiency by steps, values are outputted to the user. These values tell the user if the flaws in thermodynamic efficiency are being caused by massive pressure losses, abnormal temperature increases or decreases, or losses in the internal energies of the system, which can be caused by fuel mixtures. Essentially, The Perfect Engine computer program specifically describes how efficient any four-stroke internal combustion engine is, and informs the user of possible solutions to increase engine efficiency and performance, thereby minimizing troubleshooting time and money.

References

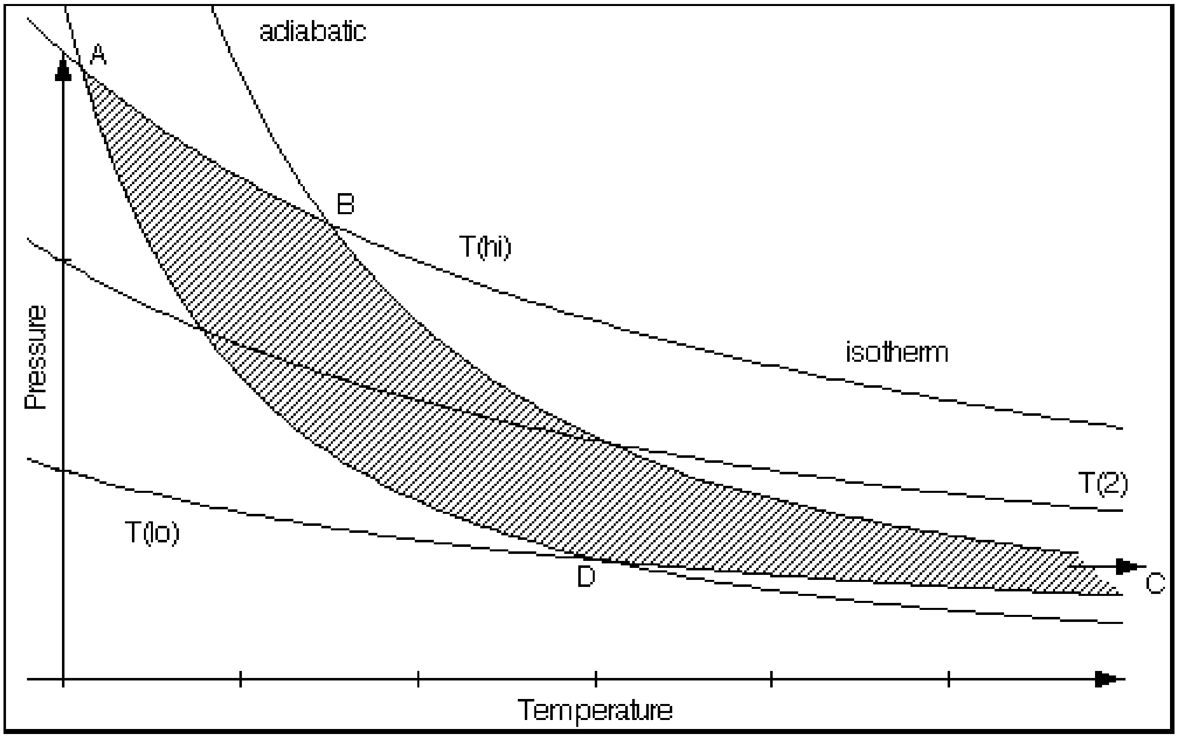
1. Moran, M.J. and Shapier , H.N., “Fundamentals of Engineering Thermodynamics:, John Wiley and Sons, 1988, Appendix A, Table A-13E
2. Chilton’s Manual - Datsun 240-z, 1982 edition, page 72.
3. Chilton’s Manual Specification Log – Table A –13E, Thermodynamics of Engine types. Page 376.
4. webphysics.ph.msstate.edu/JC/library/13-pp

Appendices

1. Carnot Cycle
2. Otto Cycle
3. Program in C++
4. Program in Excel
5. Interpolation Chart

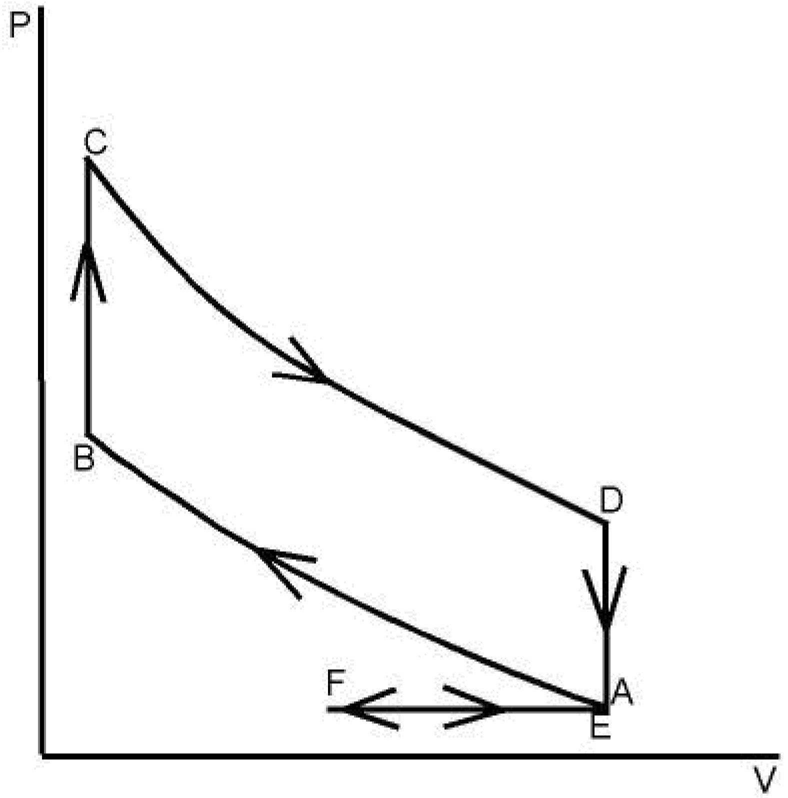
Appendix 1

Carnot Cycle



Appendix 2

Otto Cycle



Appendix 3

Program in C++

```
//James Hill and Daniel Shelley
//Period 7
//5-4-02
//The Perfect Engine

//Libraries
#include <iostream.h>
#include <math.h>
#include <stdlib.h>
#include <fstream.h>

//function prototypes
double hot(double);
double cold(double);

//finding number of variables in chart.dat
int num = 111;

int main ()
{       //opening main

//arrays
double T[num];
double Pr[num];
double U[num];
double Vr[num];

//declare variables
double V2;      //volume at ideal gas state
double V3;      //Volume at hottest Temp
double V4;      //Volume at isentropic expansion
double r;       //compression ratio
double Tb;      //Beginning Temp
double Th;      //Hottest Temp
double max;     //exact hottest temp
double min;     //exact lowest temp
double max;     //exact hottest temp
double min;     //exact lowest temp
double Ta;      //Actual Temp
double T4;      //Temp at isentropic expansion
double minV;    //lowest volume of intervals
double maxV;    //highest volume of intervals
double p1;      //staring atmospheric pressure
double p2;      //pressure of atm at first compression
double p3;      //pressure of atm at second compression
double p4;      //pressure of atm at isentropic expansion
double U1;      //Internal energy at lowest temp
double U2;      //Internal energy at isentropic compression
```

```
double U3;	//Internal energy at hottest temp
double U4;	//Internal energy at isentropic expansion
double TE;	//Thermal Efficincy

fstream InFile("chart.dat",ios::in);
if (InFile.fail())
	{	//open if loop
if (InFile.fail())
	{	//open if loop
	cout<<"File Don't work!!  Moron!!"<<endl;
	return (0);
	}	//close if loop

//stores information into Arrays
else
{	//open else loop
	for (int i=0; i<=num; ++i)
	{
	InFile>> T[i];
	InFile>> Pr[i];
	InFile>> U[i];
	InFile>> Vr[i];
	cout<<T[i]<<"    ";
	}	//close for loop
}	//close else loop
}	//close else loop

cout<<" "<<endl;

//get limits
cout<<"Enter the starting temp of the Engine:"<<endl;
cin>>Tb;
cout<<"Enter the hottest tempature your engine can take:"<<endl;
cin>>Th;

//function calls
min = cold(Tb);
max = hot(Th);

//matching volume with min and max Temps
for (int k=0; k<=num; ++k)
	{	//opening for loop
	if ((min>=T[k])&&(min<T[k+1]))
		{	//open if loop
 {	//open if loop
		minV = (((min-T[k])/(T[k+1]-T[k]))*(Vr[k+1]-Vr[k]))+T[k];
		U1 = (((min-T[k])/(T[k+1]-T[k]))*(U[k+1]-U[k]))+U[k];
		}	//close if loop
	}	//close for loop

for (int k=0; k<=num; ++k)
	{	//opening for loop
	if ((max>=T[k])&&(max<T[k+1]))
		{	//open if loop
		maxV = (((max-T[k])/(T[k+1]-T[k]))*(Vr[k+1]-Vr[k]))+T[k];
		U3 = (((max-T[k])/(T[k+1]-T[k]))*(U[k+1]-U[k]))+U[k];
```

```
        V3 = (((max-T[k])/(T[k+1]-T[k]))*(Vr[k+1]-Vr[k]))+Vr[k];
        }       //close if loop
    }       //close for loop

//getting compresion ratio
cout<<"Enter the compresion ratio"<<endl;
cin>>r;

//Isentropic compression
V2=minV/r;

for (int h=0; h<=num; ++h)
    {       //open for loop
    if ((V2<=Vr[h])&&(V2>Vr[h+1]))
        {       //open if loop
        Ta = (((V2-Vr[h])/(Vr[h+1]-Vr[h]))*(T[h+1]-T[h]))+T[h];
        U2 = (((V2-Vr[h])/(Vr[h+1]-Vr[h]))*(U[h+1]-U[h]))+U[h];
        }       //close if loop
    }       //close for loop
if (Ta<=Th)
    {       //open if loop
    cout<<"I hope you have money becasue you just melted your
engine."<<endl;
 }      //close if loop

else
{       //open else loop
cout<<"Tempature after Isentropic compression is "<<Ta<<" R."<<endl;

cout<<"Enter the starting atmospheric pressure:"<<endl;
cin>>p1;

//finding pressure at the ideal gas state
p2 = p1*(Ta/Tb)*r;

cout<<"The first pressure at ideal gas state is "<<p2<<" atm."<<endl;

//finding pressure at ideal gas state
p3 = p2*(Th/Ta);
cout<<"The second pressure at the ideal gas state is "<<p3<<"
atm."<<endl;

//matching volume and internal energy with temp
for (int k=0; k<=num; ++k)
    {       //opening for loop
    if ((Th>=T[k])&&(Th<T[k+1]))
        {       //open if loop
        U3 = (((Th-T[k])/(T[k+1]-T[k]))*(U[k+1]-U[k]))+U[k];
        V3 = (((Th-T[k])/(T[k+1]-T[k]))*(Vr[k+1]-Vr[k]))+Vr[k];
        }       //close if loop
    }       //close for loop

//Isentropic expansion
V4 = V3*r;

for (int h=0; h<=num; ++h)
```

```
  {     //open for loop
        if ((V4<=Vr[h])&&(V4>Vr[h+1]))
                {     //open if loop
                T4 = (((V4-Vr[h])/(Vr[h+1]-Vr[h]))*(T[h+1]-T[h]))+T[h];
                U4 = (((V4-Vr[h])/(Vr[h+1]-Vr[h]))*(U[h+1]-U[h]))+U[h];
                }     //close if loop
        }     //close for loop

cout<<"The temperature after Isentropic expansion is "<<T4<<" R."<<endl;

//finding pressure at isentropic expansion
p4 = p1*(T4/Tb);

//finding Thermal Efficiency
TE = (1-((U4-U1)/(U3-U2)))*100;

cout<<"The thermal efficiency of your car is "<<TE<<" percent."<<endl;
}     //close else loop
return (0);
}     //close main

//function definition for hot
double hot(double x)
{     //opening function
//arrays
double T[num];

fstream InFile("chart.dat",ios::in);

for (int i=0; i<=num; ++i)
        {     //opening for loop
        InFile>> T[i];
        }     //closeing for loop
for (int j=0; j<=num; ++j)
        {     //open for loop
        if ((x>=T[j])&&(x<T[j+1]));
                {     //open if loop
                x = (((x-T[j])/(T[j+1]-T[j]))*(T[j+1]-T[j]))+T[j];
                }     //close if loop
        }     //closing for loop
return (x);
}     //close hot funtion

//function definition for cold
double cold(double x)
{     //opening function

//arrays
double T[num];

fstream InFile("chart.dat",ios::in);

for (int i=0; i<=num; ++i)
        {     //open for loop
        InFile>> T[i];
        }     //close for loop
```

```
for (int j=0; j<=num; ++j)
        {       //open for loop
        if (T[j]>=x);
                {       //open if loop
                x = (((x-T[j])/(T[j+1]-T[j]))*(T[j+1]-T[j]))+T[j];
                }       //closing if loop
        }       //closing for loop
return (x);
}       //closing function
```

Appendix 4

Program in Excel

Tb = 540R Starting Tempurature
Th = 3600R Hottest Tempurature
Compression Ratio = 8
Starting Pressure= 0.785398163atm
0
b= 1
k= 2220.66099(spring constant)

	Isentropic Compression	Adiabatic Copression	Isentropic Expansion	Adiobatic Expansion
0	540	0	333.0991485	540
0.5	498.8949476	-162.3017871	307.7434856	498.8603045
1	381.8376618	-299.8945983	235.5366667	381.7846372
1.5	206.6490535	-391.8311755	127.4715255	206.6060153
2	3.3079E-14	-424.1150082	2.04048E-14	1.32592E-05
2.5	-206.6490535	-391.8311755	-127.4715255	-206.5772975
3	-381.8376618	-299.8945983	-235.5366667	-381.6785819
3.5	-498.8949476	-162.3017871	-307.7434856	-498.6524792
4	-540	-5.19604E-14	-333.0991485	-539.7000833
4.5	-498.8949476	162.3017871	-307.7434856	-498.583247
5	-381.8376618	299.8945983	-235.5366667	-381.5726123
5.5	-206.6490535	391.8311755	-127.4715255	-206.4912905
6	-9.9237E-14	424.1150082	-6.12144E-14	-3.97556E-05
6.5	206.6490535	391.8311755	127.4715255	206.4625397
7	381.8376618	299.8945983	235.5366667	381.4665784
7.5	498.8949476	162.3017871	307.7434856	498.3755169
8	540	1.03921E-13	333.0991485	539.4003332
8.5	498.8949476	-162.3017871	307.7434856	498.3063434
9	381.8376618	-299.8945983	235.5366667	381.3607051
9.5	206.6490535	-391.8311755	127.4715255	206.3766295
10	1.65395E-13	-424.1150082	1.02024E-13	6.62225E-05
10.5	-206.6490535	-391.8311755	-127.4715255	-206.3478457
11	-381.8376618	-299.8945983	-235.5366667	-381.2546926
11.5	-498.8949476	-162.3017871	-307.7434856	-498.0987084
12	-540	-1.55881E-13	-333.0991485	-539.1007496
12.5	-498.8949476	162.3017871	-307.7434856	-498.0295936
13	-381.8376618	299.8945983	-235.5366667	-381.1489156
13.5	-206.6490535	391.8311755	-127.4715255	-206.2620321
14	-2.31553E-13	424.1150082	-1.42834E-13	-9.266E-05
14.5	206.6490535	391.8311755	127.4715255	206.2332154

Appendix 5

Interpolation Chart