Our Polar Problem

New Mexico Adventures in Supercomputing Challenge
Final Report
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Team 044

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Executive Summary

For a brief moment, imagine your are the captain of an American Submarine during the height of the cold war. Heading towards the North Pole for a top secret reconnaissance mission, a bad situation comes up. You find that your electrical equipment has failed, including the GPS equipment and communications devices. Within a matter of hours your submarine could be far off course, with no way to contact your base. How do you find your destination with only a traditional compass (which may be no good since the magnetic north pole is nearly 900 miles away from the true North Pole!)?

Simple, you have the Navy purchase state-of-the-art real-time declination angle calculating software developed by Team 044!

With all joking and hypothesizing aside, our project's goal is to find a solution to the problems encountered in navigating with respect to the magnetic North Pole. Specifically, its aim is to find the corrective angle between the destination, the ship, and the magnetic north pole, known as the angle of declination.

So far, attempts at using spherical trigonometry equations have failed for unknown reasons. One equation provided by an online web site was unsuccessful; an equation derived by our group from three-dimensional trigonometric identities also failed.

We are now looking at a new method of coordinate-system changes that will produce a corrective angle. Essentially, it involves rotating the globe such that the ship is rotated to the North Pole, and new coordinates are calculated for the Magnetic North Pole and Destination. This rotation is necessary because when spherical coordinates are translated to polar coordinates (and then rectangular coordinates) the spherical surface is naturally distorted; however, the angle at the North Pole is the same (imagine looking at the top of a globe—ninety degrees longitude actually looks like ninety degrees).

Statement of Problem

As previously mentioned, our problem is calculating the angle of declination between a destination point, a moving ship, and the magnetic North Pole on Earth's surface.

Traditional attempts at using planar trigonometry fail because of the curvature of Earth's surface.

Description of Method Used To Solve

As of now, our program does not work. However, listed below are the two initial methods used, followed by the one we are currently trying to implement.

Method 1

- Initially we found the following equation that used three dimensional trigonometry:

  http://www.bohra.net/archive/qibla_calc.html

$$Q = \tan^{-1} \frac{\sin (L_d - L_m)}{[(\cos F_d) \times (\tan F_m)] - [(\sin F_d) \times (\cos L_d - L_m)]}$$

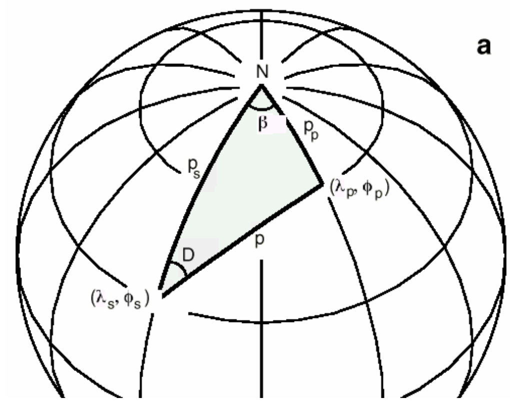
Where:

F_d = Latitude of Desired Location
L_d = Longitude of Desired Location
F_m = Latitude of Mecca (+21.45° north of Equator)
L_m = Longitude of Mecca (-39.75° east of Prime Meridian)

- We incorrectly assumed that the coordinates of Mecca could be replaced with the coordinates of the magnetic North Pole and a correct angle would be produced. It did not work for two reasons
  1. "Desired Location" refers to the vertex of the declination, or the ship in our problem, not to the destination.
  2. This equation is built around the True North pole as a reference point (note that the user only inputs his location; the angle is calculated from the True North pole, himself, and Mecca.)

Method 2

- Discovering, though not understanding spherical trigonometry, we attempted to derive our own formula for calculating the angle of declination given the following formula:



$$\cos p_p = \cos p_s \cos p + \sin p_s \sin p \cos D$$

λ_s = LAP = Latitude of Plane

ϕ_s =LOP = Longitude of Plane

λ_p =LAM = Latitude Magnetic North Pole (82.3)

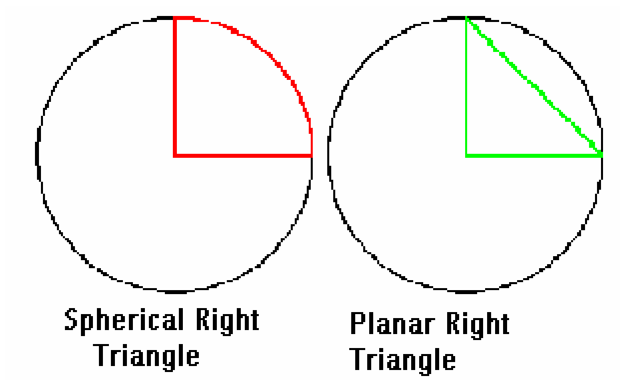
ϕ_p = LOM = Longitude Magnetic North Pole (113)

- Solving for D should give the angle of declination:
- Declination = $\tan^{-1}$[sin |LOP - LOM|

  [(cos LAM) x (sec LAP)] - [(sin LAM) x cos |LOP - LOM|]]

- Unfortunately, it didn't work; it gave seemingly random numbers from 1 to 5.

<u>Method 3</u>



- Say the left circle above is a bird's eye view of the globe, with a spherical right triangle from degrees 0,0 to 90,0 to 90, 90. Say the right circle is merely a right triangle drawn on paper with a circle superimposed over it.
- Although both triangles are very different (one exists in three dimensions, the other in two), note a striking similarity. Both accurately portray a ninety-degree angle in the center.

Essentially, our third method takes advantage of this similarity. We first rotate the ship's location to the North Pole, so that the angle of declination is also at the North Pole. Then, converting spherical coordinates to cylindrical coordinates and then to rectangular coordinates gives us an undistorted view of the declination angle, which can then be calculated using simple trigonometry.

Results

Unfortunately, we have not developed a working program yet. As previously stated, our first two attempts failed. We are still writing the code for our third method. Updated results will be coming soon.

Conclusions

We found that spherical trigonometry is a difficult concept to grasp, one that is perhaps beyond our current level of knowledge.

We also found that coordinate-system conversions are a fairly simple way to bypass complicated spherical trigonometry, and should prove to be successful.

Most Significant Original Achievement

Our most significant original achievement has been our third method of converting coordinate systems. It saves a lot of time and unnecessary calculations.

Acknowledgements

We would like to recognize and thank Mr. Schum for giving us the idea of a declination program while we were struggling to come up with ideas.

We would like to thank Mr. Matthews for practical insight and advice given while working on our project.

Finally, we would like to thank Mr. Cordwell for his suggestions of coordinate-conversions; without his help we would have been stuck far behind.