Dynamic Wave Modeling

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Team 102
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E.0 Executive Summary

The purpose of this project, Dynamic Wave Modeling, is to develop a set of computational models and computer programs that are based on calculations (developed by the team) designed for evaluating unique solutions of the wave equations. Wave phenomena involve the transmission of energy and momentum by means of vibratory impulses through various types of matter, and in the case of hydrodynamic waves (as evaluated in this study), specifically through a liquid medium.

This project evaluates the reactions of waves in a defined environment (sea water) in terms of wave dynamics that can be described by applied physics. Computational programs are created to investigate these characteristics in terms of defined input parameters. The basis of this study is to investigate and develop real-time solutions for wave motion in the defined medium. The output data is dependent upon the dimensional characteristics of the input data, which are then used to define the wave dynamics.

Applied programs are developed that describe and permit both position and time dependent wave behavior using a unique relationship between two dependent arguments of the sine function (as shown in Section 3.0 and Appendix I); and employ working mathematical models, developed by the team, that describe the dynamics of the wave. The computer programs (written in Gnuplot and Dark Basic) display a real time presentation applying values calculated using the equations developed in Section 3.0 to describe the motion of a wave throughout several different timeframes; although as developed in this study by "normalizing" to the density of the transport medium the equations can be made applicable to a large range of input data (see Appendix I for a complete table of possible input density factors).

The results of the hand calculations (discussed in Section 4.0) verify the computer calculations. The output of the three functions evaluated ($y(x,t)$, Kinetic Energy (KE) and Momentum) demonstrate the flexibility and capability to perform sophisticated calculations involving these three independent parameters and allow the user to evaluate wave behavior. The equations used are modified by normalizing the equation to a fixed volume; thereby allowing the equation to be evaluated on a density basis. The equations in this study use water (simply to demonstrate the mathematical models); however the user can input any density factor to describe wave dynamics traveling through any medium.

1.0 Introduction

1.1 Purpose

The purpose of this project, Dynamic Wave Modeling, is to develop a set of computational models and computer programs that are based on calculations developed by the team designed for evaluating unique solutions of the wave equations. Wave phenomena involve the transmission of energy and momentum by means of vibratory impulses through various types of matter, and in the case of hydrodynamic waves (as evaluated in this study), through a liquid medium. Energy may be transmitted through all states of matter (gas, liquid, or solid) by longitudinal movement of particles. This report investigates the wave motion in a particular media, specifically a liquid; the basic equation is modified to permit the medium used for transmitting the wave to be applied for a variety of transport systems.

1.2 Scope

This project develops a computational model that has the capability of describing a specific set of solutions for specified wave conditions; the mathematics and the physics that describe the mechanics of a wave are evaluated through parametric simulations of the wave. The kinetics of the wave are evaluated, specifically the momentum and energy. Using selected input parameters; wave behavior is analyzed for a range of input values. The density of the transport medium (such as sea water) remains constant, for all intents and purposes, in the calculations.

Simple progressive waves can be described as a combination of both longitudinal and transverse motion. In longitudinal waves, such as waves in a liquid medium, the particles that are in vibratory motion move back and forth in a direction parallel to the course of propagation of energy. Similarly the transverse wave motion (which is described by the $y(x,t)$ in Section 3.0) defines vertical displacement of the wave as a function of both down range position, $x(t)$, and time (t).

The project is designed to promote a better understanding relating to the physical aspects of the mechanics in a hydrodynamic wave traveling through a liquid medium. The following conditions are identified and reviewed to define the characteristics:

- Kinetic energy of the wave
- Wave characteristics (velocity, wave mass, wave height, wave length)
- Density of medium (sea water)

The wave function is modeled using frequency, period, wave height, and wave mass.

1.3 Computer Application

Two computer programs are utilized by the team members for the purpose of calculating, evaluating, and displaying in real-time motion the dynamics of the physical behavior of the wave. These are: Dark BASIC and Gnu Plot, for the purpose of calculating, evaluating, and displaying in real-time motion the dynamics of wave behavior. These languages can perform sophisticated calculations and handle a large amount of variables. Functions are created in order to perform various calculations involving computational data collection. The evaluations and compiled output data represents the behavior of the wave as a function of both downrange position and time (refer to section 3.1). The computer program employs models that incorporate a variety of information that describes a general wave traveling through a specific transport medium. This equation can then be modified by adapting the input parameters to allow the user to calculate wave behavior in a defined media, in the technique employed in this study, the density of the transporting medium is used as the input parameter.

2.0 Project Proposal

2.1 Project Description

This project evaluates the reactions of waves in a defined environment (sea water) in terms of wave dynamics that can be described by applied physics. Computational programs are created to investigate these characteristics in terms of defined input parameters. The basis of this study is to investigate and develop real-time solutions for wave motion in a specified medium. The output data is dependent upon the dimensional characteristics of the input data, which are then used to define the wave dynamics.

Through mathematical and physical investigation of the characteristics of the defined wave, a computer program is developed to:

- Identify required input variable
- Methodology of selecting input parameters
- Evaluation output information
- Creating visual, real-time, representations of the models

Dynamic Wave Modeling examines the behavior of a wave as it travels through a specified medium, such as water, as a result of the applied forces. By changing the input parameters, the results outputted by the programs can be applicable to solid, liquid, of gas environments. This project evaluates one specific transport medium, sea water; however, by changing the input parameters, other media can be evaluated.

The fundamentals mathematic presentation assumes a standard wave mechanics solution involving two variables in the argument of a typical transcendental function (refer to Section 3.1). This approach allows the user to manipulated the input data to the transcendental function and thereby develop a unique time-dependent solution; specifically one parameter can be held to a constant value and the second parameter allowed to change over a wide range of values. This process can be reversed, that is, holding the second parameter constant and allowing the first to change (a selected output solution is presented in the appendices).

3.0 Analytical Methodology

3.1 Mathematical Bases

Applied programs are developed that describe and permit both position and time dependent wave behavior using a unique relationship between two dependent arguments of the sine function (as shown in the equations below); and employ working mathematical models, developed by the team, that describe the dynamics of the wave. The computer programs display a real time presentation applying values calculated using the equations developed in this section to describe the motion of a wave throughout several different timeframes; although, as developed in this study by “normalizing to the density of the transport medium,the equations can be made applicable to a range of input data.

The basic equations used to model the wave are;

- $y(x,t)=A\sin 2\pi\left(x(t)/\lambda - t/T\right)$

which can also be written as:

- $y(x,t)=A\sin 2\pi\left(x(t)/\lambda - V/\lambda * t\right)$

where:

- y(x,t)=vertical poison of the wave at a given horizontal distance down range (X axis) as a function of time
- x(t)=horizontal distance down range (X axis)(user defined)
- λ=wavelength
- t=given time (user defined)
- T=period of wave
- V=wave Velocity (down range)
- A=arbitrary Constant (user defined)

The next term required is the vertical (transverse) velocity of the wave for a specified downrange distance and time, which is found by taking the derivative, with respect of time, of the y(x,t) function:

- $v_y=dy(x,t)/dt$

which equals:

- $v_y=-(A*V)(2\pi/\lambda)\cos\left[2\pi/\lambda*\left(x(t)-V*t\right)\right]$

With the v_y term, the transverse kinetic energy and momentum of the wave can now be evaluated:

- KE=1/2**M* v_y^2
- P=M* v_y

where:

- KE=kinetic energy of the wave (Joules)
- M=mass of the medium in a specified volume (Kg)(density*volume)

The kinetic energy (KE) and momentum (P) functions, for purposes of these analyses, are normalized to volume using the relationship that mass=density*volume.

- KE/Vol=1/2*ρ*v_y^2
- P/Vol=ρ*v_y

where:

- ρ=density

These equations are now able to calculate the motion of a wave traveling through any medium simply by changing the density factor.

The equations are developed to show wave action as a function for both real time and downrange position; and specifically enable the user to present real time representation of wave behavior through specified timeframes as a function of the energy and momentum relationships.

These equations are modeled by applying a transcendental function (the initial study utilizes a sine function to initiate the calculation process). The argument of the sine function is modeled to describe the wave as a function of distance and time, velocity and wavelength are also included in the argument. An initial set of equations are evaluated to define the wave action. A range of variables are selected for each input parameter to enable the program to examine and evaluate a variety of outputted wave functions (the actual values selected are presented in the appendices).

3.2 Computer Program

With this computer program, an enhanced knowledge of wave behavior is quickly evaluated and easily accessed. The program allows users to input a variety of variables to describe wave behavior involving loss of energy due to friction between the wave and the surfaces over which the wave is acting, such as the ocean floor acting upon a water wave, or the surrounding gasses attenuating the wave action. This program uses the mathematical equations defined in Section 3.1 to create a representation of a wave through a defined media. The input parameters used in these equations (velocity, density, mass, ect.) are defined as variables in the computational programs to enhance the flexibility of the program.

This project uses two programming languages to evaluate the wave action defined above. The computer programs are:

- Dark BASIC – This program models a three-dimensional wave and displays the results as a function of real time motion. To accomplish this, matrices are modeled by setting the maximum height of a vertex in the matrix to the output of our y(t) function. Because the function is a sine function, a wavelike effect is created.
- Gnu Plot – This language calculates the sine equation used to model a wave and outputs the results in a three-dimensional mesh. This program can upload source code from .dat and .txt files.

4.0 Results

4.1 Computer Calculations

The calculations performed by the computer programs allow the users to create a realistic representation of a dynamic wave traveling through a defined medium. Because of the calculational techniques applied to this program, careful attention to each component of the overall procedure is required in order to ensure correct representation. To ensure the validity of the results received from the computer programs, identical calculations were performed by hand. The results of both sets of calculations are compared, and after the computational results were verified to be correct, the representation is proven to be correct (see appendices for complete list of calculations).

5.0 Conclusions

5.1 Mathematical Models

Through a process of examination, calculation and graphical representation, the physics of wave dynamics is evaluated and outputted visually in real-time motion based on a double variable argument for the transcendental functions.

5.2 Computer Program

The computer programs model the overall dynamics of a wave traveling through a defined medium, which can be defined with a variety of specified inputs. The particular inputs utilized in this project represent water (refer to Appendix 1 for a complete set of density variables). The computer program is written to allow the user to model a wave traveling through a defined medium simply by inputting a density factor that is unique for the medium. This aspect of the program is verified by the computer program output

The transcendental functions model wave behavior utilizing two users input dependent variables. The validity of this solution is verified by two sets of calculations holding one variable constant for each case (calculations are shown in Appendix 1). The resultant functions allow the user to model wave behavior by selecting specified input parameters. This is confirmed by the equations and calculations shown in section 3.0 and appendices. The results of these computational programs clearly show that this technique is viable procedure for evaluating wave mechanics in a given medium.

5.3 Results

The results of the hand calculations (discussed in Section 4.0) verify the computer calculations. The output of the three functions evaluated ($y(x,t)$, KE and P) demonstrate the flexibility and capability to perform sophisticated calculations involving these three independent parameters and allow the user to evaluate wave behavior. The equations used are modified by normalizing the equation to a fixed volume; thereby allowing the equation to be evaluated on a density basis. The equations in this study use water (simply to demonstrate the mathematical models); however the user can input any density factor to describe wave dynamics traveling through any medium unique to a specified material.

5.4 Recommendations for Future Work

To improve the project beyond what has been accomplished to date, it is recommended that loss of energy term be included into the calculations to more accurately represent wave behavior over time. As developed, the equations are already written to be able to calculate wave behavior traveling through different media.

References

1. Thurman, Harold V., Introductory Oceanography, Columbus: Bell & Howell, 1975.

Appendix I

A.1.1 Basic Wave Equation

The simplest solution to the basic wave equation is given by:

$y(x,t) = A \sin [k(x – vt)]$

where k is an arbitrary constant but is also unique for the particular application, v is wave velocity, and t is time at which the downrange displacement is considered. Hence, the above equation describes the wave vertical displacement and the displacement as a function of downrange position, x, and time, t, that the wave propagation is traveling with velocity, v.

The equation for y(x,t) then describes the properties of the waves, in this, the sinusoidal behavior of the wave (this description of the wave is selected for this study) therefore describes the displacement of the wave at every point relevant to the downrange wave motion. In other words, this equation shows the dependence of the vertical position of the wave on a downrange position, x, at some time, t.

Equation $y(x,t) = A \sin [k(x – vt)]$ characterizes a typical sinusoidal wave.

Where the maximum vertical displacement, A, occurs at a distance of λ between the two consecutive points of vertical displacement. This horizontal displacement is referred to as the wavelength λ. The amplitude of the wave, A, occurs at the moment at those intervals where the sine takes on the values of +1 or –1; then the value of A at these points is the amplitude of the wave and represents the maximum displacement of the wave referenced to the equilibrium position. If a maximum displacement occurs at the downrange position x_1, the general y(x,t) equation can be written as:

$y(x,t) = A \sin \{k[x-vt]\}$

and rewriting so that the equation is a general expression:

$y(x,t) = A \sin \{k[x(t)-v*t]\}$

The next item is to determine an expression for the constant k. This can be accomplished as follows:

- substitute $y(x,t) = A$ and $x(t) = x_1$
- rewrite the general y(x,t) as follows,

$A = A \sin \{k[x_1 – v*t]\}$
Or

$1 = \sin\{k[x_1 – v^*t]\}$

The sine function is identically equal to 1 when the argument of the sine function is 90° or $\pi/2$, therefore

$k(x_1 – v^*t) = \pi/2$

Recall that λ is defined as the wavelength, then the next maximum value occurs at

$x(t) = x_1 + \lambda$

and subsequent maximum values occur exactly 2π radians. Substituting

$k[x(t) - v^*t] = k[x_1 + \lambda - v^*t]$

and since

$k[x(t) – v^*t] = \pi/2$

then $k[x(t) + \lambda - v^*t] = \pi/2 + 2\pi$

because λ occurs every 2π radians.

The constant k can be found by subtracting the above equations thus:

$k[x(t) – v^*t] = \pi/2$
$k[x(t) + \lambda - v^*t] = \pi/2 + 2\pi$

substitute for $\pi/2$

$k[x(t) + \lambda - v^*t] = k[x(t) – v^*t] + 2\pi$

$k\, x(t) + k\, \lambda - k\, v^*t = k\, x(t) – k\, v^*t + 2\pi$

$k\, \lambda = 2\pi$
or $k = 2\pi/\lambda$

and the final expression for y(x,t) can be written as:

$y(x(t),t) = A \sin\{2\pi/\lambda[x(t) – v^*t]\}$

This is the general equation describing the vertical displacement of the wave as a function of the two variables, x(t) and t, expressed as an argument of the sine function. The term v is the wave velocity and for this development is considered a constant; however a series of solutions can be developed for a family of values of wave velocity if desired. The above equation of y(x(t),t) can be written as follows:

$y(x(t),t) = A \sin \{2\pi[x(t)/ \lambda - t/T]\}$

by substituting for v*t = t/T where T is the wave period.

Based on the equation for y(x(t),t), the vertical velocity of the wave at any downrange location and time can be found by taking the derivative of the y(x(t),t) function which results in the vertical velocity of the wave as described by the two variable components of the wave, x(t) and t, can be evaluated for any downrange position and time (this derivation is included in the following appendix, Appendix II). Combining the respectable solution for y(x(t),t) and v(x(t),t); the wave momentum and energy can be determined as a function of downrange position and time (Appendix II).

As a final point, the analysis in Appendix II is modified by substituting for the mass using the relationship:

mass = density * volume

and by assuming a constant volume, then density can be substituted for mass,

$m = \rho * v$
or $\rho = m/v$

(Refer to the table below to view densities of other materials)

and by assuming a fixed volume for purposes of analysis, the density term can be substituted for mass in the momentum and energy equations – these solutions are given in Appendix II. By replacing the mass term with density term, the solutions for displacement, momentum, and energy can be tailored to a variety of materials; in fact, a set of parametric solutions can be generated using density as the variable. The next appendix generates solutions for two input variables – water and air – to demonstrate this point.

TABLE OF DENSITIES OF THE MORE COMMON MATERIALS

To convert kg/m^3, multiply by 10^{-3}

Material	kg/m^3	lb/ft^3
Aluminium	2691	168
Asphalt	1506	94
Brass	8394	524
Brick, common	1794	112
Bronze	8715	544
Cement, Portland	1506	94
Cement, Portland (set)	2483	155
Chalk	2195	137
Clay (wet)	3124	195
Coal (anthracite)	1554	97
Coal (bituminous)	1346	84
Coke	1202	75
Concrete masonry	2323	145
Copper ore	4197	262
Corn (bulk)	593	37
Glass	3140	196
Gravel	1922	120
Iron, cast	7209	450
Kerosene	817	51
Lead	11342	708
Limestone	2739	171
Manganese ore	3204	200
Mortar, rubble	2483	155
Nitrates (loose)	1602	100
Oils, mineral	929	58
Paper	929	58
Petroleum, crude	881	55
River mud	1442	90
Sand	1602	100
Sandstone	1442	90
Steel	7769	485
Tin	7337	458
Zinc	7049	440

Appendix II

Sample Calculations
Wave Equation with Two Arguments

The following calculational results are for a standard wave equation uniquely developed to describe the use of two variables in the argument of the sine function describing both the vertical displacement and downrange position of a particle in the wave proper. Several expressions can be used to model wave behavior, these are shown at the end of this paper, the equation selected for this particular study is given as follows:

$$y(x,t) = \sin[2\pi / \lambda (x(t) - v * t)]$$

where $\lambda =$ wave length (cm)
x(t) = downrange position (cm)
v = travel velocity of wave in x-direction (cm/sec)
t = time of observation (sec)

and since this is the sine function, the maximum value occurs each time the argument of the sine, that is, [2π/λ(x(t)-v∗t)] Ξ |π/2| or when the sine function Ξ |1| (Note: In this application the value of the sine function is allowed to vary between +1 and -1). A constant "A" can be inserted in to the equation to define the amplitude of the sine function, thus:

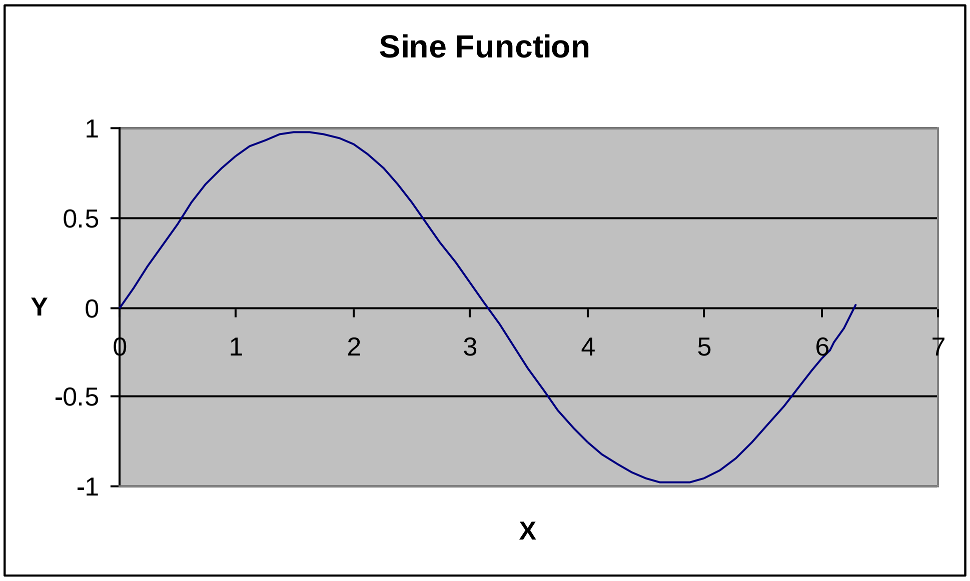
$$y(x,t) = A\sin[2\pi / \lambda (x(t) - v * t)]$$

Substituting representative values for the sine parameters, the y(x,t) equation can be solved for two conditions based on the argument of the sine. The first condition, Condition 1, is for a fixed downrange position; and the second condition, Condition 2, is for a fixed time. Sample calculations are given below (a complete set of results are given in the computer output appendix):

Condition 1: assume v = 100cm/sec, x(t) = 500 cm, λ = 200 cm

x(t)	t	y(t) for A=1, x(t) = 500cm
500	0.0	0.00
500	0.1	0.31
500	0.4	0.95
500	0.5	1.00
500	0.6	0.95
500	1.0	0.00
500	1.1	-0.31
500	1.5	-1.00
500	1.7	-0.81
500	2.0	0.00

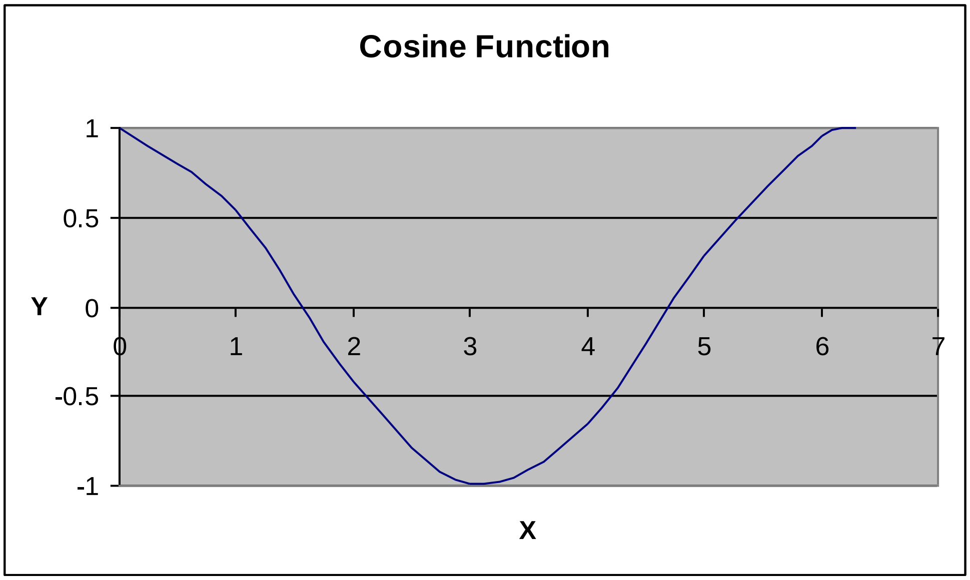
Plot:y(t) for fixed downrange position (constant x position:



Condition 2: assume t =1.0 sec, v = 100 cm/sec, λ = 200 cm

t	x(t)	y(x(t)) for A=1, t=1.0 sec
1.0	0	0.0
1.0	10	-0.31
1.0	20	-0.59
1.0	40	-0.95
1.0	50	1.00
1.0	60	-0.95
1.0	80	-0.59
1.0	100	0.0
1.0	150	1.0

Plot y(x) for fixed time:



Example calculations for both variables using selected input parameters:

λ = 200 cm
x(t) = 100 cm
v = 100 cm/sec
t = 0.6 sec

Using these input conditions, the general equation can be evaluated as follows:

$$y(x,t) = A\sin[2\pi/\lambda\,(x(t) - v*t)]$$

and substituting in the specified values for the argument of the sine:

$$y(x,t) = A\sin[2\pi/200\,(100 - 100*0.6)]$$
$$y(x,t) = A\sin[\pi/100\,(100 - 60)]$$
$$y(x,t) = A\sin[\pi/100\,(40)]$$
$$y(x,t) = A\sin[0.4\pi]$$
$$y(x,t) = A(0.95)$$

As a result, using this technique to describe a wave function, a large sample field can be constructed using a range of input values for λ, x(t), v, and t; these parameters can be sued to define the wave for any set of input parameters desired. The computer program developed for this study performs the indicated functions and displays the results in a time dependent presentation. Thus, with the above equation using two variables for the argument of the transcendental function a wide variety of results can be obtained. The wave model used for this presentation is a sine function with the argument modified to show the two variables; however other functions could be used to describe the input conditions as desired. A follow-on set of calculations in a separate part of the appendix shows the equations for both the momentum of the wave and the kinetic energy of the wave. Further, the equations in that section have been normalized to the volume of the sample and the calculations then based on the density of the material through which the wave is traveling; this enables the user to model a large variety of materials without adjusting the mathematics of the equation for changing sample dimensions.

<u>Variations of the basic wave equation:</u>

$$y(x,t) = \sin[2\pi/\lambda\,(x(t) - v*t)]$$
$$y(x,t) = \sin[k*x(t) - \omega t]$$
$$y(x,t) = \sin[2\pi\,(x(t)/\lambda - f*t)]$$
$$y(x,t) = \sin[2\pi\,(x(t)/\lambda - t/T)]$$

where λ is the wave length, f is the wave frequency, T is the wave period. The time units used for this study are in seconds. Also, the first and fourth equations listed above are the basic equations applied to the computer program to analyze wave behavior.

Appendix III

A.3.1 Gnuplot Program

Gnuplot is a mathematical plotting platform that uploads source code from .txt and .dat files. It operates on common languages such as Basic, C++, Java, and 4tran. Gnuplot was incorporated into this investigation to provide a more comprehensible wave representation. That is, one wavelength can be viewed of the defined wave.

v=50

v defines wave velocity

q=200

q defines wavelength

t=1

t defines given time

All input variables are assigned a value above.

set xrange[0:q]
set yrange[-1:1]

Above, the range of the x and y scales are set. Note that the x range is scaled from 0 to q, with q being the defined wavelength.

splot 1*sin((2*3.14159)*((x/q)-(v/q)*t))

The splot command directs the program to plot the function in all three dimensions. It achieves this by projecting a 2D rendering into a 3D mesh rendering.

A.3.2 Dark Basic Program

Dark Basic is a Basic language compiler commonly utilized as a gaming platform. For our needs, we have adapted Dark Basic to model complex mathematics. The purpose of this adaptation is to provide real-time modeling of user-defined waves. As a note, in Dark Basic, screen positions are measured from the upper right corner of the screen. Thus, the upper right corner location is (0,0).

Prior to actual modeling, the screen display must be established:

`*****

REM SCREEN SETUP

Please note that “Rem” and “`” statements are only remarks used to organize the program code. They have no affect on program operation.

set display mode 1024,768,32

The “set display mode” command sets the screen to 1024 pixels wide, by 768 pixels tall, by 32 pixels “deep”.

sync on : sync rate 50

Sync rate is the speed that the screen refreshes itself in a given time, in Dark Basic, the time is measured in seconds. The above command line turns on the sync, and sets a rate of 50 refreshes per second.

autocam off

In Dark Basic, “autocam” is an automatic camera setting that allows the computer to control the camera. With this setting “on” all objects in the program will be visible. For our purposes, we want to be in full control of our camera, and so “autocam” has been turned off.

hide mouse

After preparing the screen, input from the user must be provided to generate the wave.

REM USER INPUT

set cursor 350,25
Input "Enter Wave Constant: ",A#

set cursor 350,100
Input "Enter Horizontal Position: ", Xt#

set cursor 350,175
Input "Enter Wavelength: ",q#

set cursor 350,250

```
Input "Enter Time Period:  ",Tt#

set cursor 350,325

Input "Enter Given Time:  ",t#
```

Please note that the "set cursor" command will be frequently used in the program. This program sets the simple text cursor to a certain point on the screen (x,y position). In the following "Input" command, this is where the text will appear. The "Input" command consists of the Input declaration, followed by text to be displayed on the screen, and lastly, separated by a comma, the variables' name.

```
REM GENERATE 3D WATER SUBSTANCE

rem generate a plain texture
cls rgb(32,192,255)
get image 3,0,0,32,32

rem Size of the water matrix
matrixSizeX#=100:matrixSizeZ#=100
nsquarex=64:nsquarez=48
tileSizeX#=matrixSizeX#/nsquarex:tileSizeZ#=matrixSizeZ#/nsquarez

rem Create the tank, and texture it with a caustics texture
make object box 2,matrixSizeX#,matrixSizeZ#/2,matrixSizeZ#
scale object 2,-100,-100,-100
position object 2,0,(matrixSizeZ#/-4),0
load image "caustics.bmp",2:texture object 2,2
rem These variables are used for the movement of the caustics
scru#=0.001:scrud#=-0.00001
scrv#=-0.0005:scrvd#=0.000005

rem Create the water matrix
make matrix 1,matrixSizeX#,matrixSizeZ#,nsquarex,nsquarez
position matrix 1,matrixSizeX#/-2,0,matrixSizeZ#/-2
dim waves(nsquarex,nsquarez)
for i=0 to nsquarex
  for j=0 to nsquarez
    waves(i,j)=(rnd(72))*5
  next j
next i

rem Texture the water matrix
load image "water.bmp",1
```

```
prepare matrix texture 1,1,nsquarex,nsquarez
n=0
for j=nsquarez-1 to 0 step -1
  for i=0 to nsquarex-1 step 1
    inc n
    set matrix tile 1,i,j,n
  next i
next j
ghost matrix on 1

rem Create the tank, and texture it with a caustics texture
make object box 30,matrixSizeX#,matrixSizeZ#/2,matrixSizeZ#
scale object 30,-100,-100,-100
position object 30,0,(matrixSizeZ#/-4),100
texture object 30,2

rem Create the water matrix
make matrix 20,matrixSizeX#,matrixSizeZ#,nsquarex,nsquarez
position matrix 20,matrixSizeX#/-2,0,45
`matrixSizeZ#/-5
dim waves(nsquarex,nsquarez)
for ik=0 to nsquarex
  for jk=0 to nsquarez
    waves(ik,jk)=(rnd(72))*5
  next jk
next ik

rem Texture the water matrix

prepare matrix texture 20,1,nsquarex,nsquarez
n=0
for jk=nsquarez-1 to 0 step -1
  for ik=0 to nsquarex-1 step 1
    inc n
    set matrix tile 20,ik,jk,n
  next ik
next jk
ghost matrix on 20
```

In the above section, matrix sizes are defined, along with their subdivisions, and matrices are created. The matrices are then textured using **for – next** loops.

```
REM SETUP CAMERA

rem Camera initialization
camdist#=matrixSizeZ#:bearing#=90:azimuth#=300:camchanged=1
```

The above line of code defines the distance between the camera and matrices using the matrix depth.

```
REM MAIN LOOP
do

REM DECLARE CONSTANTS

P#=3.14159
s#=10
vol#=1000000
dens#=1.2*vol#

REMSTART *** VARIABLES
A#=wave amplitude
Xt#=horizontal position
q#=wavelength
v#=velocity
t#=given time
Tt#=period
vol#=water volume (cc's)
dens#=density...1.2 gcc's
REMEND
```

All constants and variables are defined and listed above. The constants are assigned a value, while most variables are user – defined in above sections.

```
REM VELOCITY FUNCTION

v#=(-1*(A#/Tt#))*(cos(2*3.1415)*((Xt#/q#)-(t#/Tt#)))

REM TIME FUNCTION

yt#=A#*sin(2*P#)*((Xt#/q#)-((v#/q#)*t#))

REM KINETIC ENERGY FUNCTION

KE#=0.5*dens#*(v#*v#)

REM MOMENTUM FUNCTION

Mo#=dens#*v#
```

All functions operating in the program are defined above. All operate on user input.

```
REM TEST FUNCTIONS
`yt#=A#*sin(2*P#)*(t#/s#)
`yt#=sin(30)+2
```

Above are two functions used to quickly test program changes without the complete user based input functions.

```
REM MATRIX MODELING

randomize matrix 1,yt#
randomize matrix 20,yt#
```

Randomize commands define the maximum height a vertex on the matrix can be, either positive or negative. Since output from a sine function is being used, a water like effect is achieved.

```
rem Update matrix
update matrix 1
update matrix 20
```

Update matrix commands “refresh” the matrices to their new randomized value.

```
REM WATER EFFECT

  rem Scroll the caustics texture
  scru#=scru#+scrud#:if (scru#<=-0.001 and scrud#<0) or (scru#>=0.001
and scrud#>0) then scrud#=scrud#*-1
  scrv#=scrv#+scrvd#:if (scrv#<=-0.001 and scrvd#<0) or (scrv#>=0.001 and
scrvd#>0) then scrvd#=scrvd#*-1
  scroll object texture 2,scru#,scrv#
  scroll object texture 30,scru#,scrv#
```

Above are the commands used to scroll the textures on the sides of the containers. It provides a more realistic water effect.

```
REM CAMERA POSTITIONS

  rem Move and zoom the camera with mouse
  movex=mousemovex():movey=mousemovey()
 ` movex=leftkey() : movexa=rightkey() : movey=upkey() :
moveya=downkey()
  if movex<>0 or movey<>0 or movexa<>0 or moveya<>0
    azimuth# = azimuth# + movey
    bearing# = wrapvalue(bearing# + movex/2)
    if azimuth#<280 then azimuth#=290
```

```
    if azimuth#>350 then azimuth#=360
    camchanged = 1
  endif
  click=mouseclick()
  if click>0
    if click=1 and camdist#>matrixSizeZ#/2.00 then dec
camdist#:camchanged = 1
    if click=2 and camdist#<matrixSizeZ#*2.00 then inc
camdist#:camchanged = 1
  endif
```

The above commands define the camera movement in relation to mouse movement.

```
  if A#<100
  if upkey()=1 then A#=A#+1
  endif
  if A#>1
  if downkey()=1 then A#=A#-1
  endif

  if inkey$()="a" then position camera 75,3,120 : point camera 0,0,120

  if returnkey()=1
  set matrix wireframe on 1
  set matrix wireframe on 20
  else
  set matrix wireframe off 1
  set matrix wireframe off 20
  endif
```

Above are the commands for real time user input. While the program is running, the constant A can be redefined between 1 and 100. However, the user can define a higher value in the input section described earlier. If higher than 100, the value of A can be decreased while the program is running. The matrices can also be viewed in a wire frame state and at matrix level.

```
  rem If the camera moved, recalculate its position
  if camchanged=1
    camx# = camdist# * sin(azimuth#) * cos(bearing#)
    camz# = camdist# * sin(azimuth#) * sin(bearing#)
    camy# = camdist# * cos(azimuth#)
    position camera camx#,camy#,camz#
    point camera 0,0,0
    camchanged = 0
  endif
```

Above are the functions to calculate the camera position in regard to the matrix.

REM END LOOP

```
  set cursor 50,10
   print "yt#: ";yt#
  set cursor 904,10
   print "Velocity: ";v#
  set cursor 50,50
   print "Kinetic Energy: ";KE#
  set cursor 512,50
   print "Momentum: ";Mo#
  set cursor 904,50
   print "Amplitude: ";A#
  set cursor 50,738
   print "Hold Return Key to view wireframe"
  set cursor 350,738
   print "Hold A to view matirx at water level"
  set cursor 650,738
   print "Press up or down arrowkeys to change wave constant"
```

The above print commands print all the output from the programs functions, and display the values on the screen.

```
 ` sleep 500
  sync

loop
```