

# Data-Based Approach to Estimating Ice-Shelf Melt Rates

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# 1 Executive summary

Climate change and rising temperatures are causing increased melting of ice in the polar regions. But the melt rate of the ice shelves of Antarctica is difficult to directly measure as most of it happens not on the surface, but under the ice by the interaction of ice and sea water. The problem addressed in this project is to apply a data-based approach for predicting the ice melt rate from an Antarctic ice shelf based on ocean water measurements made at the front of the ice shelf. Specifically, data recording instruments placed at the ice-shelf front take measurements of the temperature, velocity, and direction of the sea water flow. With the information they obtain, a melt rate can be calculated by comparing the heat of the water going in and out of the ice shelf. The heat difference, which by the law of preservation of energy has to be converted into a different type of energy, goes for converting ice into melt water. We use data for the Amery Ice Shelf derived from the Amisor project of the Australian Antarctic Data Centre. The challenges we face are dealing with defective and incorrect measurements as well as with the low spacial resolution (having a small number of instruments placed on an ice shelf front stretching over hundreds of miles). We develop and analyze several methods for dealing with such challenges. We replace missing or incorrect values by existing values that are closest in time and space and we use Voronoi diagrams to assign each point of the ice shelf front to the closest point with valid measurements. Our prediction for the annual melt is 46.85 gigatons and is consistent with the estimates of other studies that use alternative method and/or data. Our method allows estimating daily values and seasonal variations of the ice melt amounts.



Figure 1: Ice shelves are sheets of ice stretched above the ocean’s surface formed by the flow of glacier down and over the ocean surface.

## 2 Introduction

Containing global warming and climate change and predicting their consequences is a major challenge of the 21st century. One of the risks is that increased temperatures will melt much of the snow and ice that cover the south and north polar regions. This will cause the global sea level to rise and create floods in major cities across the globe. Oceans are rising 3.2 millimeters each year [7], and the rate is accelerating. Accurately estimating the amount of melt rates at different locations is important, but challenging problem. Antarctica contains the largest body of ice on the Earth, and its ice is melting constantly. If all of the ice in Antarctica melts, the ocean level could rise an extra 60 meters [1].

The continent is surrounded by *ice shelves* (Figure 1), which are bodies of ice that stretch out over the water and most of the melting happens on the bottom surface of these shelves, directly into the ocean. Ice shelves in Antarctica cover an area of 1.561 million km<sup>2</sup>, comparable in size to the Greenland [8]. But estimating the rate of the

melting of the ice shelves in Antarctica is difficult to directly measure as it happens under the ice by the interaction of ice and sea water (Figure 2a), where it is hard to place data collecting instruments. The air temperatures in Antarctica are too low for significant amounts of ice and snow to melt on the surface. In contrast, in the Arctic and in Greenland, Figure 2b, the air temperature can become high enough so that ice and snow melt happens mostly on the surface.

The problem addressed in this project is to model and predict the ice melt rate from an Antarctic ice shelf based on ocean water measurements made at the front of the ice shelf. Specifically, data recording instruments need to be placed at the ice-shelf front that could take measurements of the temperature, velocity, and direction of the sea water flow. With the information they obtain, a melt rate can be calculated by comparing the heat of the water going in and out of the ice shelf, Figure 3. The heat difference, which by the law of preservation of energy has to be converted into a different type of energy, goes for converting ice into melt water. The idea is that if we are able to estimate that difference, we could evaluate the amount of ice melted.

Previous approaches to estimating ice-shelf melting rates include glaciological studies [5, 8], using numerical models [4], and using satellite and radar measurements [2]. Closest to our approach is the study of Herraiz-Borreguero et al. [3], but they complement sea water measurements with previous analysis and knowledge about the water currents under and around the ice shelves. Specifically, they consider modified Circumpolar Deep Water (mCDW), Dense Shelf Water (DSW), and Ice Shelf Water (ISW) currents information.

In contrast, our approach relies only on analysis of the sea water velocity, direction, and temperature at the ice shelf front. It also allows temporal resolution, i.e., to estimate the daily melt rates, while [3] produced a single annual total melt prediction



(a) Antarctic ice shelf extending over the ocean. Downloaded from <https://www.cnn.com/2015/05/16/us/antarctica-larsen-b-ice-shelf-to-disappear/index.html>.



(b) Stream of melted ice on Greenland downloaded from <https://www.businessinsider.com/greenland-approaching-threshold-of-irreversible-melting-2019-1>.

Figure 2: Comparison between ice melting in the south and north polar regions.

only.

Our goal is to make estimations of the ice melted for each day of the year and the total ice melted as well as to analyze seasonal variations. The challenges we face are to find data that contains measurements over an entire shelf front and to deal with defective and incorrect measurements as well as with the low spacial resolution (having a small number of instruments placed on an ice shelf front stretching over hundreds of miles). The specific ice shelf we study is the Amery Ice Shelf, the third largest ice shelf in Antarctica.



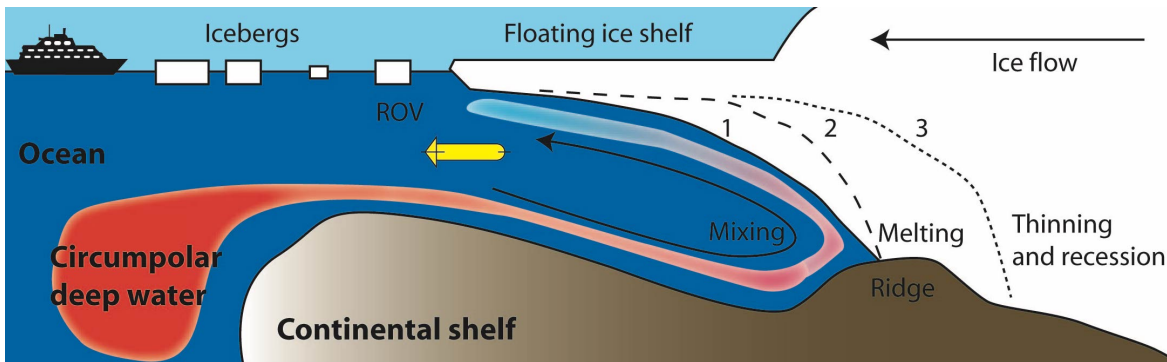


Figure 3: Diagram of ice melting caused by sea water under an ice shelf. Downloaded from <http://www.antarcticglaciers.org/2014/05/west-antarctic-ice-sheet-collapsing/>.

## 3 Methods

### 3.1 The data set

The first step is to find the right data. It takes some effort to find suitable measurements, since they should be from an ice-shelf front, from a time period of at least a year of continuous measurements, and with sufficient spacial density. For instance, many measurements are taken from the ice shelf interior, rather than from its front. Some studies collect data from instruments attached to seals, but seals cannot be limited to stay at the ice shelf front. Finally, the data set should include measurements of the velocity, direction, and temperature of the sea water, all three taken at the same times.

We eventually came across the Australian Antarctic Data Centre (AADC) collection, which has data files collected by their moorings at the Amery Ice Shelf front in Antarctica. Amery Ice Shelf is suitable for our approach because of its long and narrow shape, see Figure 4. This helps the model to be more accurate, because the shelf's front is relatively short (about 230 km) compared to its area (roughly 62,000 km<sup>2</sup>), making the placement of instruments closer together and permitting more accurate estimates. Amery is the third largest ice shelf in Antarctica and the largest in East Antarctica.

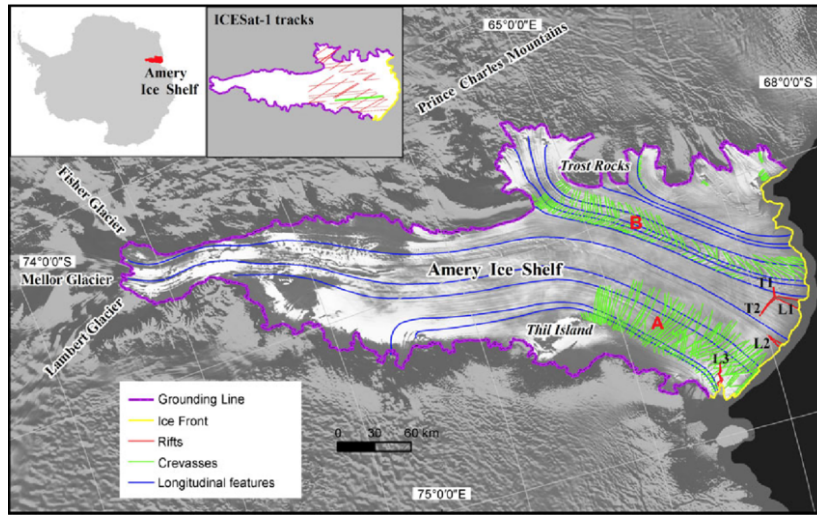
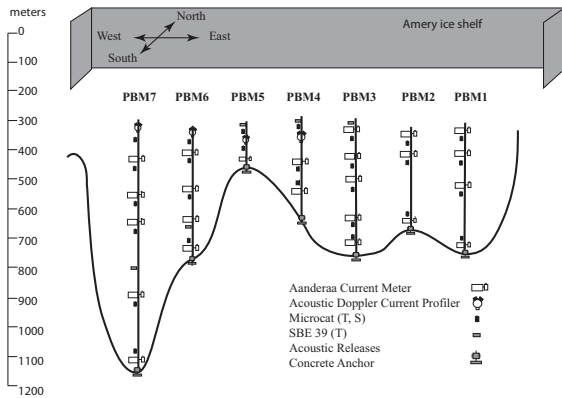


Figure 4: Satellite map of Amery ice shelf. Downloaded from [https://www.researchgate.net/figure/Sketch-map-of-the-characterized-Amery-Ice-Shelf-overlaid-on-MOA-image-The-green\\_fig1\\_262571908](https://www.researchgate.net/figure/Sketch-map-of-the-characterized-Amery-Ice-Shelf-overlaid-on-MOA-image-The-green_fig1_262571908).

The instruments are attached to seven moorings, whose positions are illustrated on Figure 5a. Moorings are long wires that connect the ocean’s floor to the ocean’s surface and which have data instruments attached to them that collect different measurements like temperature, velocity, and salinity. Moorings are dropped by ships, as one can see on Figure 5. The anchors to the moorings must be drilled securely into the ocean floor with precision so that they do not shift.

The moorings were deployed by a ship in February 2001 and collected by another ship a year later, in February 2002. It is relatively expensive to send ships to South Antarctica and there is a very small window of time during the austral summer when the area is free of the thick ice and accessible, which explains why there are only few such data sets available none of them very recent one.

The moorings held two types of devices that recorded the measurements we needed. The first is the RCM type instrument, which records depth, direction, position and



(a) Diagram of seven moorings at ice shelf front [3].



(b) A ship dropping moorings at an ice shelf front. Downloaded from <https://pbs.twimg.com/media/Deen9BEU8AAUr8q.jpg>.

Figure 5: Data is collected by placing mooring at the front of the ice shelf.

velocity, plus other parameters that we don't use (Figure 6a), and which allows us to estimate the amount of water going pass the moorings. Their data files contain about 200,000 lines in total, corresponding to a frequency one record per every hour.

The second instrument type, the microCAT type, measures temperature and salinity (Figure 6b), which will be used later to estimate the amount of heat going in and out of the shelf and the melt rate. The microCAT files had more than 2.5 million lines, much more than the RCM type, because each instrument was set to record measurements every 5 minutes.

```

MOORING : amisor1
INSTRUMENT : Aanderaa rcm 7837x
POSITION : 69 22.014 S 074 38.153 E
INSTRUMENT DEPTH : 735 (m)
INSTRUMENT PRESSURE : 744.0 (dbar)
BOTTOM DEPTH : 750 (m)
MAGNETIC DECLINATION APPLIED: -76.46 (deg)
COMMENT : temperature data bad for whole record; pressure data suspicious
COMMENT :
COMMENT :
COMMENT :
MOORING : amisor7
INSTRUMENT : microcat 1119
POSITION : 68 28.659 S 070 23.118 E
INSTRUMENT DEPTH : 695 (m)
INSTRUMENT PRESSURE : 703.5 (dbar)
BOTTOM DEPTH : 1135 (m)
COMMENT :
COMMENT :
COMMENT :
COMMENT :
decimal_time date and time(UTC) speed dir E comp N comp press temp cond
      yyyy mm dd hh mm (cm/s) (deg.true) (cm/s) (cm/s) (dbar) (deg.C) (mS/cm)
31.02083333 2001 02 01 00 30 -999.00 -999 -999.00 -999.00 -999.00 -999.00 -999.000
31.06250067 2001 02 01 01 30 -999.00 -999 -999.00 -999.00 -999.00 -999.00 -999.000
31.10416800 2001 02 01 02 30 -999.00 -999 -999.00 -999.00 -999.00 -999.00 -999.000
31.14583534 2001 02 01 03 30 -999.00 -999 -999.00 -999.00 -999.00 -999.00 -999.000
31.18750267 2001 02 01 04 30 -999.00 -999 -999.00 -999.00 -999.00 -999.00 -999.000
. . .
46.60441694 2001 02 16 14 30 1.10 341 -0.36 1.04 670.76 -999.00 -999.000
46.64608428 2001 02 16 15 30 1.10 352 -0.15 1.09 670.76 -999.00 -999.000
46.68775161 2001 02 16 16 30 1.10 5 0.10 1.10 663.77 -999.00 -999.000
46.72941895 2001 02 16 17 30 2.54 17 0.74 2.43 670.76 -999.00 -999.000
46.77108628 2001 02 16 18 30 2.13 7 0.26 2.12 663.77 -999.00 -999.000
46.81275362 2001 02 16 19 30 1.10 357 -0.06 1.10 670.76 -999.00 -999.000
46.85442096 2001 02 16 20 30 1.10 356 -0.06 1.10 663.77 -999.00 -999.000
46.89608829 2001 02 16 21 30 1.10 344 -0.30 1.06 663.77 -999.00 -999.000
46.93775563 2001 02 16 22 30 1.10 345 -0.28 1.06 663.77 -999.00 -999.000
46.97942296 2001 02 16 23 30 1.10 352 -0.15 1.09 670.76 -999.00 -999.000
47.02109030 2001 02 17 00 30 1.20 40 0.77 0.92 663.77 -999.00 -999.000
47.06275763 2001 02 17 01 30 3.99 52 3.14 2.46 663.77 -999.00 -999.000
47.10442497 2001 02 17 02 30 4.82 55 3.94 2.76 663.77 -999.00 -999.000
47.14609231 2001 02 17 03 30 4.71 62 4.16 2.21 670.76 -999.00 -999.000
47.18775964 2001 02 17 04 30 3.68 52 2.90 2.27 663.77 -999.00 -999.000
47.22942698 2001 02 17 05 30 1.10 54 0.89 0.65 663.77 -999.00 -999.000
47.27109431 2001 02 17 06 30 1.10 50 0.84 0.71 663.77 -999.00 -999.000
. . .
49.75766421 -1.8846 27.4196 34.5364 19 02 2001 18 11 02
49.76113641 -999.0000 -999.0000 -999.0000 19 02 2001 18 16 02
49.76460861 -1.8849 27.4193 34.5363 19 02 2001 18 21 02
49.76808080 -1.8850 27.4193 34.5364 19 02 2001 18 26 02
49.77155300 -1.8853 27.4191 34.5365 19 02 2001 18 31 02
49.77502520 -1.8857 27.4190 34.5368 19 02 2001 18 36 02
49.77849740 -1.8858 27.4187 34.5365 19 02 2001 18 41 02
49.78196960 -1.8858 27.4183 34.5359 19 02 2001 18 46 02
49.78544180 -1.8860 27.4182 34.5360 19 02 2001 18 51 02
49.78891400 -1.8853 27.4182 34.5352 19 02 2001 18 56 02
49.79238620 -1.8843 27.4186 34.5349 19 02 2001 19 01 02
49.79585840 -1.8840 27.4193 34.5352 19 02 2001 19 06 02
49.79933060 -1.8841 27.4194 34.5355 19 02 2001 19 11 02
49.80280280 -999.0000 -999.0000 -999.0000 19 02 2001 19 16 02
49.80627500 -1.8841 27.4194 34.5355 19 02 2001 19 21 02

```

(a) Example of an RCM data file.

(b) Example of a microCAT data file.

Figure 6: The structure of the data files.

## 3.2 Processing the Data

### 3.2.1 Cleaning of faulty data

We need to make sure that every valid data point is included, while the faulty ones are removed. Since the instruments sometimes malfunction and record invalid data, such data elements are replaced by ‘-999.0’ in the dataset. As seen in Figures 6a and 6b, the first several hundred lines record defective values. This must be because as the moorings get dropped, it takes time for the instruments to get to function properly. Later on, most of the record are valid, but occasionally ‘-999’ may appear.

To make use of such occasional defective items of data, we replace the missing elements with values likely to be close to the real ones. An easy to implement solution is to replace each faulty value by the average of the values recorded by that instrument. For this, we used the python function SimpleImputer from the sklearn

library. However, due to seasonal variations, such strategy approximates inaccurately measurements from the summer and the winter. Hence, we implemented our own imputer function, which replaces each missing data with the closest previous value from the same instrument that is a good one. This is justified by the fact that temperature, velocity, and direction are likely to change gradually in time and there is not much difference between measurements taken at roughly the same time.

In order to check the validity of our hypothesis, we analyzed the data to estimate the error of such replacement. We go about this by taking into account the difference of measurements taken at certain time intervals and averaging them. Specifically, we loop over skip values  $s$  between 1 and 1000, and for each  $s$  we average the absolute value of the difference  $t(i) - t(i - s)$  for all valid temperatures  $t$  at times  $i$  and  $i - s$ . The results are shown on Figure 7. One can see that the average errors are small, and they increase slowly with the value of  $s$ , which means that our hypothesis is correct for the temperature data.

In the second test, we measure in a similar way the error in the approximation of the volume, which depends on both the direction and the velocity. (Computation of the volume is discussed in the next subsection.) We see here a different behavior. The error is relatively high (tens of times higher than in the temperature case), and its growth in time is very slow. This shows that replacing volume data leads to much higher error than replacing temperature data. Unfortunately, faults in volume data are also much more frequent, with about 24432 invalid elements, vs only 3480 for the temperature.

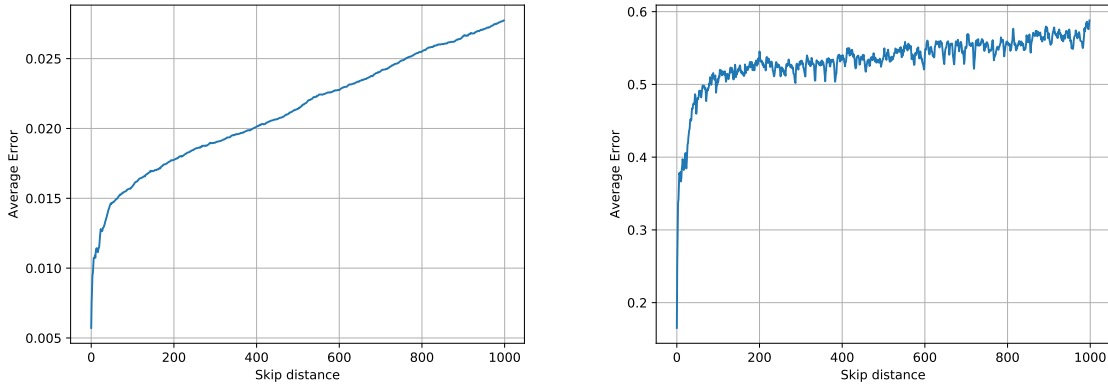


Figure 7: Error due to replacement of faulty values. (a) Temperature data; (b) Volume data. The error displayed is the absolute value of the difference between the two measurements divided by the absolute value of the original measurement.

### 3.3 Geometric aspects of the data

To use the formulas for the melt rate, we need values for the velocity and direction of sea water movement as well as the temperature at each point of the shelf front. However, we have such values only at several points (where the instruments are). For each of the remaining points, we use the value of its closest instrument location.

This means that we have to compute for each instrument a data structure, called a *Voronoi region*, which gives the set of points closest to the instrument's location. The set of all Voronoi regions makes the *Voronoi diagram* (Figure 8) of the set of points representing the instrument locations. Since the Voronoi diagram covers the points of the entire plane, the regions on the periphery are infinite. So, for the ice shelf front, we add some additional points on the periphery (which we ignore in our analysis) to remove those infinite regions.

In our Voronoi diagram, the  $x$  coordinate correspond to the distance to the be-

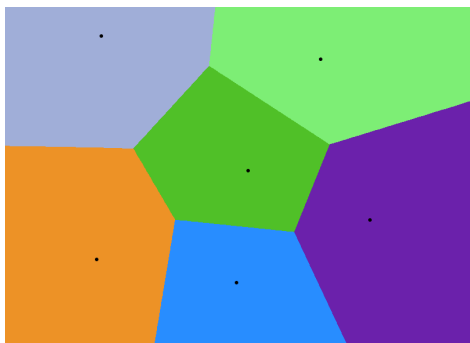


Figure 8: Voronoi diagram example. Each region is the set of points closest to one of the sites, shown as black points. Image downloaded from <http://blog.alexbeutel.com/voronoi/basic-vor.png> in February 2019.

ginning of the front and the  $y$  coordinate is the depth of the instrument. In order to find the  $x$  coordinates of the moorings, we had to find the coordinate of the beginning of the ice shelf, for which we used Google Maps. The longitude and latitude of the two moorings on either end of the front were shown visually on Google Maps and then clicked the point beside it where the ocean starts and ice ends. These specific locations, we named 'amisorA' and 'amisorB', and their locations were  $(-69.394711, 76.007650)$  and  $(-68.429035, 70.140795)$ . Then, the distance to each mooring was calculated using the Haversine formula. This formula gives the distance between two points on a sphere given their longitudes and latitudes.

As can be observed on the Voronoi diagrams shown in Figures 9, there are both colored and uncolored areas, and the uncolored ones represent the infinite regions that are actually not part of the shelf's front. If compared with Figure 5a, one can recognize the shape of the ocean floor at the bottom of the diagrams. Figure 9 (a) illustrates the average volumes of water going through the corresponding regions, with red colors meaning the water is going in the ice shelf, and blue means it is going out. Figure 9 (b) shows the average temperatures with red for warmer and blue for colder water.

Analyzing the diagrams, we observe that the top regions are much bigger than the other ones due to the fact that the top instruments are positioned relatively deep, at depth of about 500 m. There is a relatively big difference between values in adjacent regions. Those may potentially result in inaccuracy in the model. One way to deal with these issues is to readjust the areas of the Voronoi regions based on the balance of incoming and outgoing volumes of water. Although melting increases the amount of outgoing water, such increase is small fraction of the total sea water. We computed that the disbalance is about 15%, which is not a huge amount, but still shows some inaccuracy.

We first tried a machine learning model, the `LinearRegression` function of the package `sklearn`, to find optimal adjustments of the areas that will minimize the disbalance. However, this didn't work out as many of the optimal scaling coefficients (and areas) turned out to be negative. We then tried the function `Lasso`, which allows to constrain the solution to be nonnegative. This option did produce nonnegative coefficients and achieved a quite small disbalance of only 1.5%, but came with another shortcoming, more than half of the coefficients were zero, meaning that we have to ignore the majority of the data. In fact, with such modification of the areas, the melt rate predicted was three times higher than the expected range, so we had to abandon this idea.

Next we tried get a better approximation of the values for points that don't have instruments at them. The problem with the straightforward Voronoi-regions approach is that we use only the value at single instrument location to assign a value for each such point. This results in big difference between the values close to the boundaries of some of the Voronoi regions, as seen on Figure 9. In the refined approach, we first assign a value at each Voronoi vertex, which is a corner of a Voronoi region, equal to the average of the values of all Voronoi regions it belongs to. Then we triangulate



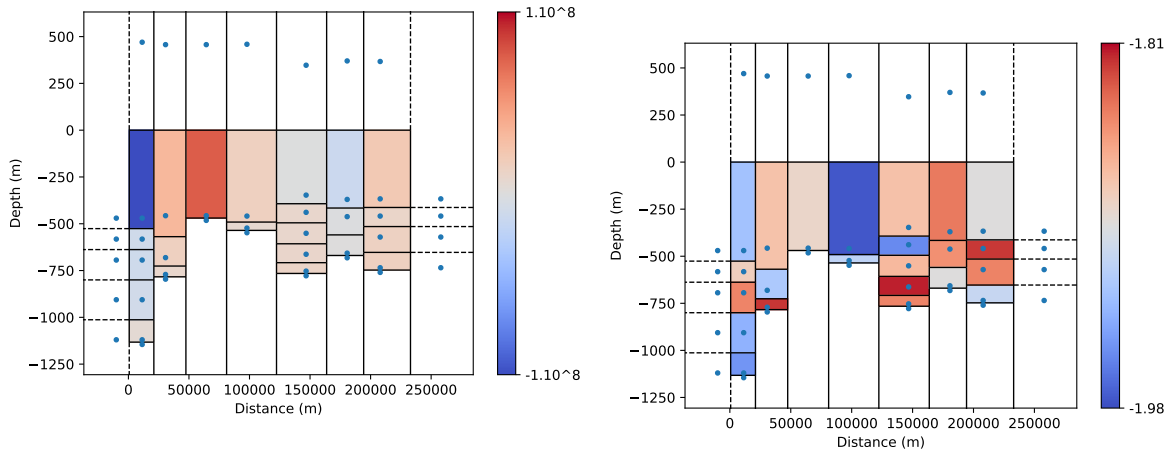


Figure 9: Voronoi diagrams illustrating average volumes (a) and temperatures (b) per region. Instrument positions are shown as blue dots.  $y$  axis gives the depth and the  $x$  axis gives the distance from the ice-shelf leftmost point, in meters.

each Voronoi region and assign value to that region equal to the average value of its vertices. The result is shown on Figure 10.

### 3.4 Computing the flux

After processing and cleaning the data, the next phase is to compute the flux, or the amount of seawater passing through the ice shelf front per second. This involves three steps. First we compute the area of each region, which is either a polygon, if the original Voronoi diagram is used, or a triangle, in the case of the smoothed variation. Then we compute the angle of the seawater movement with respect to the direction of the ice shelf front. Finally, we multiply the sine of that angle with the velocity and area of the corresponding region.

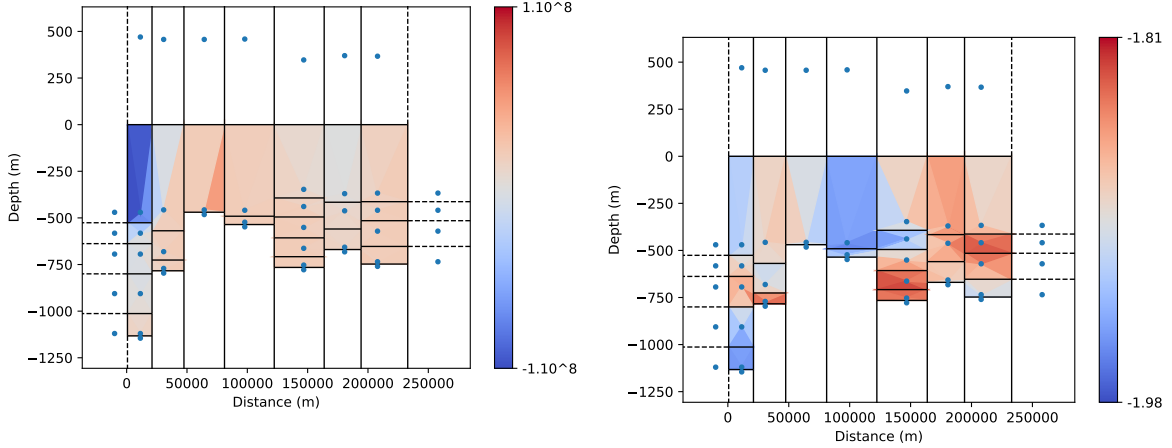


Figure 10: Voronoi diagrams illustrating smoothed average volumes (a) and temperatures (b) per region using multiple original values for computing approximations.

### 3.5 Deriving and applying the meltrate formula

Once we have estimated the flux, we can find the melt-rate by using the heat exchange formula. It gives the melt rate as a fraction, whose numerator is the heat difference of water going in and out of the ice shelf, and whose denominator is the amount of heat it takes to melt one kilogram of ice (1).

$$m_H = \frac{\text{heat in/out difference (J)}}{\text{heat per kg ice melt (J/kg)}}. \quad (1)$$

For the numerator, our computed flux is needed, the density of the water, the temperature of the the water going in and out of the shelf, and the water's heat capacity, formula (2)

$$H_{diff} = F \rho_{sw} (\theta_{in} - \theta_{out}) c_{sw}. \quad (2)$$

The denominator consists of three components, (3). Since the ice temperature is typically well below the freezing point, we need to calculate the energy it takes to warm ice to its melting point, which is proportional to the difference between the freezing point of sea water  $\theta_f$  and the measured temperature of ice  $\theta_{ice}$ , formula (4). The value

of  $\theta_{ice}$  for the Amery ice shelf was taken from [3]. The value of the ice-capacity constant and other parameters are given in Figure 11. Once the ice is at melting temperature, it needs to consume additional energy to change its state from solid to liquid, which is given by the latent heat of ice, formula (5). Finally, the water leaving the ice shelf has temperature above the freezing point, so additional energy is consumed for warming up the water, resulting in formula (6).

$$H_{melt} = H_{warm\_ice} + H_{melt\_ice} + H_{warm\_sw} \quad (3)$$

$$H_{warm\_ice} = c_i(\theta_f - \theta_{ice}) \quad (4)$$

$$H_{melt\_ice} = L \quad (5)$$

$$H_{warm\_sw} = c_{sw}(\theta_{out} - \theta_f) \quad (6)$$

Replacing (2)–(6) in (1) gives the final heat-to-melt formula (7).

$$m_H = \frac{F \rho_{sw} (\theta_{in} - \theta_{out}) c_{sw}}{c_i (\theta_f - \theta_{ice}) + L + c_{sw} (\theta_{out} - \theta_f)} \quad (7)$$

For the values of the different parameters, given in the table in Figure 11, we used information provided in [3] and [6].

We applied formula (7) using measurements for each hour of the time interval of the study, and summed up the computed volumes of melted ice in order to compute a total for the entire year.

### 3.6 Coding environment

We used for coding Python language and multiple python libraries including numpy, sklearn, scipy, and matplotlib. We implemented or adapted our own functions for computing the Haversine distance, bearings, angles, areas, for plotting, etc. Our programs contain almost 1000 lines of code. We used Canopy as a python development environment.

$F$	flux	$0.52 \cdot 10^6 \text{ m}^3 \text{ s}^{-1}$
$\rho_{sw}$	density	$1027 \text{ kg m}^{-3}$
$\theta_{in}$	temp of DSW	$-1.92^\circ$
$\theta_{out}$	temp of ISW	$-2.09^\circ$
$c_{sw}$	SW heat capacity	$4000 \text{ J kg}^{-1} (\text{ }^\circ\text{C})^{-1}$
$c_i$	Ice heat capacity	$2090 \text{ J kg}^{-1} (\text{ }^\circ\text{C})^{-1}$
$\theta_f$	freezing temp of glacial ice	$-2.769^\circ\text{C}$
$\theta_{ice}$	temp of the basal ice	$-15^\circ\text{C}$
$L$	latent heat of ice	$334,000 \text{ J kg}^{-1}$

Figure 11: Parameters used in the heat-to-melt exchange formula.

## 4 Results

### 4.1 Annual melt and flux estimates

After running our code using the smoothed Voronoi values (Figure 10), we found the water flux to be  $6.27 \cdot 10^{12}$  cubic meters per year and the amount of melted ice is 46.85 gigatons per year. In order to compute the depth of ice this corresponds to, we divide the total melt by the ice-shelf area ( $60,000 \text{ km}^2$ ) and use the density of ice ( $920 \text{ kg m}^{-3}$ ) [3] and get a value of 0.85 m of ice thickness melted per year.

Using the standard Voronoi representation (Figure 9), we compute an annual melt slightly lower, 45.5 gigatons per year. We assume the other value, 46.85 as likely to be more accurate.

### 4.2 Temporal analysis of the results

An advantage of our method is that it allows to analyze the flux and melt amounts on a daily basis. When being plotted, temperature and ice melt rates show seasonal changes. In Figure 12, the orange line represents the amount of melt for each day of the

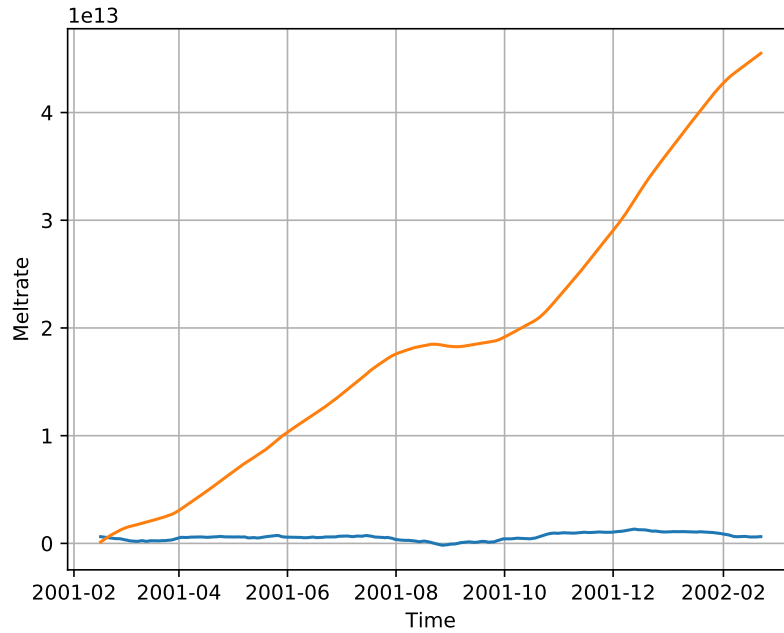


Figure 12: Plot showing melt rate during 2/2001-2/2002.

year of between February 2001 and February 2002, while the green is the accumulation of the melt until the corresponding point in time.

In order to explore the correlation between the sea water temperature and melt rate, we also plotted the temperature. Figure 13 shows the temperature variation for the period of study.

We see that the melt rate is not well correlated with the water temperature, and they both show seasonal variation. First of all, the variation in temperature is relatively small, the difference between the maximum and minimum is only  $0.035^{\circ}\text{C}$ . Note that this temperature is averaged over the entire shelf front, while there is more variation between temperatures in individual sub-regions of the front, as illustrated on Figure 9. Secondly, the warmer water going in will take some time to warm the ice to the melting point, hence there will be a delay on its effect on increasing the melt rate.

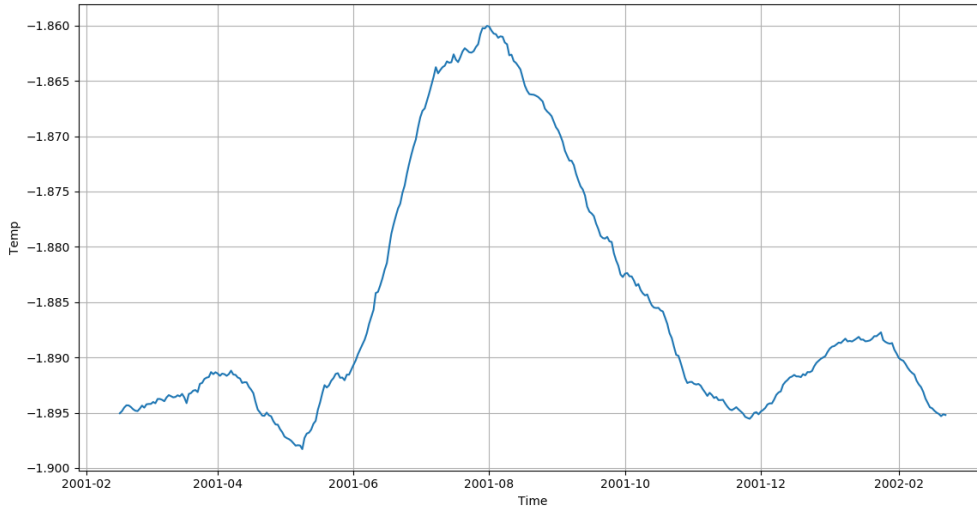


Figure 13: Plot showing average temperatures during 2/2001-2/2002.

Also, we can observe that the sea-water temperature maximum is in August, when it is summer in the northern hemisphere, but winter in Antarctica. This looks strange, but can be explained that there is delay of the warm water to get from the northern warmer regions to Antarctica and the ice shelf. The temperature of the sea water is less dependent on the sun and air temperature, and much more on the ocean currents. Similar delay of the sea water warming has been observed in [3].

### 4.3 Comparison with previous results

Other authors have used different methods in order to estimate the melt rates at the Amery ice shelf and their results are comparable to ours. Depoorter et al. [3] estimate the annual melt at 39 gigatons per year in average, while Rignot et al. [8] estimate the annual malt rate at 35.5 gigatons per year in average. Both papers use methods based on glaciological analysis, rather than sea water temperature. Yu [9] estimates

the annual rate at 27 gigatons per year using satellite and radar data. Finally, the study of Herraiz-Borreguero et al. [3], which uses the same data set as us, predicts a slightly larger amount, 57.4 gigatons per year.

All of these studies can agree that ice is melting from Amery in dangerously large amounts, putting in risk the health of the planet.

Study	Annual ice-melt amount
Here	46.85 gigatons per year
Herraiz-Borreguero [3]	57.4 gigatons per year
Deporter [2]	39 gigatons per year
Rignot [8]	35.5 gigatons per year
Yu [9]	27 gigatons per year

Figure 14: This table shows our melt rate compared to previous studies results.

## 5 Conclusion

We developed tools that allow the analysis of noisy data of sea water measurements from an ice shelf front that allow one to estimate daily and annual melt rates and applied it to data from the Amery ice shelf. Our predictions are consistent with the estimates of other researchers using different data and/or different methods.

One of the limitation of this approach is that we don't have measurements for each point of the ice shelf, but approximate large regions by measurements at a single point or a small number of points, leading to potential inaccuracies. Nevertheless, our results are quite accurate when compared to estimates given by other scientists. Another limitation is that we have to use data from 2001-2002, but more recent data

is not currently available. Finally, it is hard to positively validate the results since the real amount of ice melt is not known. However, our results are consistent with the predictions of the other studies.

The next step of this project could be computing the meltrate based on salinity. This would give alternative estimates and one can check how well they correlate with the heat-exchange predictions. The idea of such approach is that one can measure the salinity of the water going in and out and comparing how much the salinity has dropped, which can be used to give an estimate of the fresh water added as a result of the ice melt.

Another direction to continue this work is to use a data-based approach based on alternative data sources such as satellite data.

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