Entropy In Chess

New Mexico

Supercomputing Challenge

Final Report

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Team #45

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Executive Summary

Information theory deals with the quantification or storage of information, and measures information content in the form of entropy. However, despite the fact that chess is a game that has perfect information for both players and has been investigated using combinatorial game theory, our project sought to quantify information trends and different game states using information theory to gain better insight into the mechanics of chess. Toward this goal, our project has examined chess using joint entropy of the two board states, conditional entropy, and Shannon entropy. Shannon entropy, in essence, is the basic quantification of the amount of information in any given scenario, whereas conditional entropy is the amount of information one player has given the other's possible moves. Lastly, joint entropy is similar to conditional entropy except for the fact that it looks at both sides of the board from both player's perspectives. We find that these forms of entropy decrease over the course of the game and that if a particular player is winning their entropy tends to increase while the opposite is true for the losing player.

Introduction

Our project deals with entropy and, more specifically, how it relates to chess. Although none of us actually play chess competitively, Aengus' background in working with entropy in years prior allowed for us to apply a concept that we were interested in elsewhere. Thus, we came up with the idea to apply it to chess – a game that has perfect information, making it the perfect candidate. After this, we stuck with a somewhat simple question: how does entropy change over the course of a chess game? To answer this question, we applied three different kinds of entropy calculations to produce an in-depth analysis of the entropy trends throughout the course of a chess game. We achieved this by programming several Python scripts that, 1) separated all of the game files into their own respective files, 2) sorted these files by the outcomes of the games (to analyze the trends in entropy of losing vs. winning players), and, 3) calculated and graphed the bits of entropy for each game in the 2021 World Chess Championships, assuming that each move would be played with the same likelihood, or are equiprobable.

Next, we used this code as a basis for the next scripts that took into account the weighting of each move using the Stockfish chess engine. Essentially, Stockfish weighs each move in a chess game based on how beneficial it is to the player. So, if a move had a rating of 200 centipawns (essentially the equivalent of taking two pawns), whereas another move had a rating of 1000 centipawns, then the higher rated move would be far more beneficial and, in the context of these grandmaster games, far more likely to be played. Thus, we integrated Stockfish into our previously built code to develop a more realistic understanding of the trends of entropy over the course of a game.

Description

Entropy Calculations:

- Shannon Entropy was used to quantify the number of bits for a player's board state and their number of moves.

$$H_s(\mathrm{P}) = -\sum_{x \in \mathrm{X}} p(x) \log p(x)$$

- Conditional Entropy was used to describe the amount of information needed to know move Y given move X.

$$H_C(\mathrm{P}) = -\sum_{y \in \mathrm{Y}} \sum_{x \in \mathrm{X}} p(x, y) \log \frac{p(x)}{p(x, y)}$$

- Joint Entropy was used to quantify the number of bits for the board state and the total number of moves.

$$H_J(\mathrm{P}) = -\sum_{y \in \mathrm{Y}} \sum_{x \in \mathrm{X}} p(x, y) \log p(x, y)$$

Portable Game Notation file:

```
[Event "FIDE World Cup 2021"]
[Site "Krasnaya Polyana RUS"]
[Date "2021.07.13"]
[Round "1.2"]
[White "Sargsyan, Shant"]
[Black "Martinez Reyes, Pedro Ramon"]
[Result "1-0"]
[WhiteTitle "GM"]
[BlackTitle "FM"]
[WhiteElo "2626"]
[BlackElo "2417"]
[ECO "B31"]
[Opening "Sicilian"]
[Variation "Nimzovich-Rossolimo attack (with ...g6, without ...d6)"]
[WhiteFideId "13306766"]
[BlackFideId "3901807"]
[EventDate "2021.07.12"]
[EventType "k.o."]

1. e4 c5 2. Nf3 Nc6 3. Bb5 g6 4. O-O Bg7 5. c3 Nf6 6. Re1 O-O 7. d4 d5 8. e5 Ne4
9. Be3 Qb6 10. Bxc6 bxc6 11. Qc1 f6 12. exf6 Bxf6 13. Nbd2 Bf5 14. Nxe4 Bxe4 15.
Ne5 Bxe5 16. dxe5 Qb5 17. e6 Rf5 18. f3 Bd3 19. a4 Qc4 20. b3 Qxb3 21. Bxc5 Re8
22. Bxa7 Ra8 23. Bd4 1-0
```

We used three different types of entropy to examine the intricacies of entropy over the natural progression of a chess game. Shannon entropy is the first of which we used and is the simplest. It quantifies the amount of information contained within any probability distribution. In chess specifically, it measures the number of bits for an individual player's board state and their moves. Additionally, we used conditional entropy, which quantifies the amount of information needed to predict move y given move x for each move in set X and Y. We applied this to chess because move x would represent one player's move whereas move y would represent the opposing player's potential move; this is important to quantify as it allows us to see how much information is needed by each player and how, when that differs, it affects the outcome of the game. Lastly, we applied joint entropy. It quantifies the entirety of the information from a given board state and every move from each player. This, similar to conditional entropy, allowed us to examine the effect both players had at all times and the trends in entropy that came with.

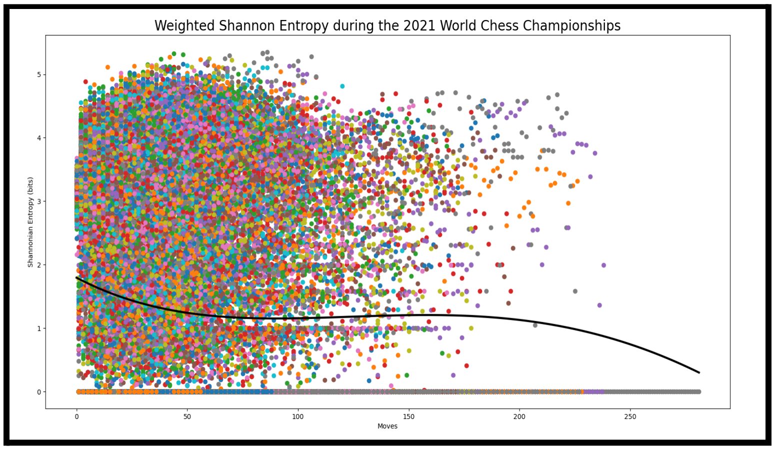
Essentially, we coded several – eight in total – Python scripts that separated the game files – PGN (portable game notation) files – and sorted them based on the outcome of the game. Then several of our scripts calculated the entropy for each move of each game, both with equiprobable odds and with the weighted determinations made by Stockfish. As previously stated, the first step we took was to separate each individual game into its own PGN file; we had to do this because the data we got was a single PGN file that contained all the games played during the 2021 World Chess Championship. With this done, we then wrote a program to sort the games into categories based on the outcome of said game – the ones in which white won, black won, and draws – providing us the context needed to be able to more closely analyze how entropy changes throughout the game and how it differs because of varying outcomes. We accomplished this by using the metadata within each PGN file (as seen above) to retrieve the outcome of the game; despite the fact that this was a relatively simple process, it was incredibly beneficial nonetheless and allowed us to gather a deeper insight into how and why entropy changed within these different contexts.

With that, we then wrote several Python programs that performed the three equations (as seen above) to calculate Shannon entropy, conditional entropy, and joint entropy. Subsequently, we did this again using the framework of Stockfish's weighting system for moves in a chess game. We did this to get a deeper understanding of the probabilities of moves throughout a game which granted us a more realistic perspective of the moves being played – as opposed to assuming that each move would have an equal benefit to the player if played.

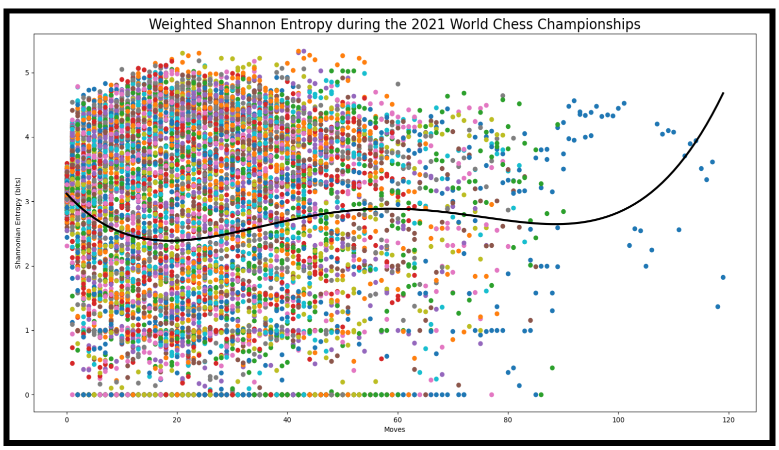
Stockfish, to clarify, quantifies each possible move in centipawns, as stated previously, by predicting what the next 10 moves are after the current hypothetical move and determining if that course of action would be actually beneficial to the player. Thus, if a move has a higher rating than another, it means that it has a greater likelihood to benefit the player and, in turn, just a generally higher probability to be played – which is why we used this in our revamped Python scripts to obtain the aforementioned authentic perspective.

Results

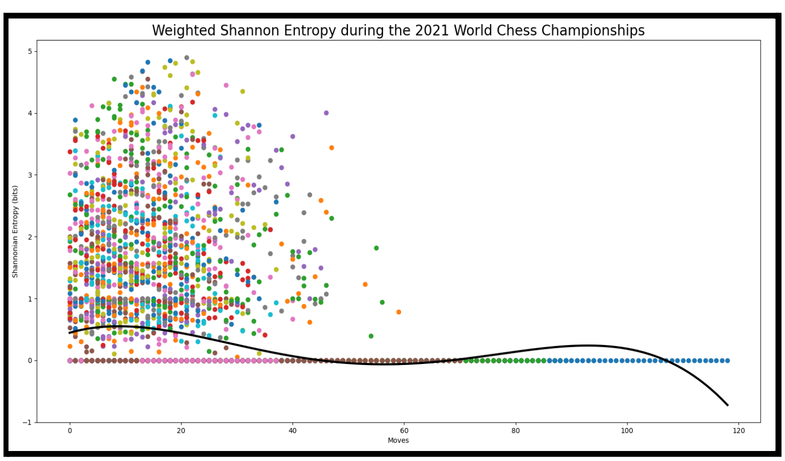
Graphs of weighted Shannon entropy of all 592 games



Graphs of weighted Shannon entropy of 197 games where white wins, from white's perspective



Graphs of weighted Shannon entropy of 197 games where white wins, from black's perspective



In the course of our study, we discovered that, generally, entropy seems to fluctuate somewhat while the game progresses due to the back and forth nature of grandmaster play, but tends to decrease overall because of pieces being taken out of the game. This results in less possible moves overall and less options for the players themselves which is the same as less information on the board as a whole; this is clearly depicted in the graph of the Shannon entropy of all 592 games processed. On the contrary, entropy specifically for the winning player seems to trend upwards as they have far more moves that they can play, again meaning they have more information overall; this can be seen in the graph of Shannon entropy of the 197 games – out of the total 592 – in which the white player won, specifically graphing the entropy of the white player's board state. Of course, this means that the losing player's entropy dramatically decreases as a result of having fewer possible moves than their opponent, depicted in the graph of Shannon entropy of the same priorly mentioned 197 games, instead calculating entropy of the black player's board state.

Conclusion

From the results we gathered, we have determined that, generally, over the course of a chess game, entropy tends to decrease. This shows us that within an isolated system like that of chess, entropy would actually trend downwards because of the removal of things like, in this case, pieces (variables) containing information. Thus, we could assume that within a different kind of isolated system in which more variables are added the entropy would trend upwards because of the addition of new vessels of information. With our newfound insights because of our results, we could also potentially alter our code to integrate lower levels of play that account for the fact that the best moves aren't always being played – thus gaining even more insight into how entropy changes within an imperfect system. Additionally, we could also apply our newfound knowledge to creating an algorithm that would attempt to predict the outcome of a chess game based on the entropy trends that it would calculate.

Overall, we have gained a significant amount of knowledge into how entropy changes based on the outcome of a chess game, the moves played during it, and how these all correlate to the natural progression of a chess game. As stated, we could apply this knowledge to creating more advanced algorithms in order to predict the outcomes of chess games or to create a chess bot that could potentially go up against humans. Regardless, this research has proven to be a great test of our coding abilities and has provided a meaningful way to build off of the concepts that Aengus introduced to us at the conception of this project.

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