Offshore, Nearshore and Foreshore Behavior of Tsunami Waves

New Mexico High School
Adventures in Supercomputing Challenge
Final Report

April 24, 2006

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Acknowledgements

The authors wish to acknowledge the following individuals for the guidance provided in the selection of a suitable topic, development of the computer code, adaptation to the computer based analyses, review, and understanding of the results, and assistance provided in the overall preparation of this report:

- Mrs. Peggy S. Larish -Teacher, Silver High School Advanced Computers Studies
- Mr. Berry F. Estes – Nuclear Engineer (Retired), Sandia National Laboratories
- Mr. Harold O. Mofjield – Oceanographer, Pacific Marine Environmental Laboratory/NOAA

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E.0 Executive Summary

The purpose of this study is to examine the properties of a tsunami wave and present the mathematical models of the wave based on the dynamics of a tsunami that is applicable to promoting an understanding of the energy of a tsunami.

This report is designed to promote a better understanding of a Tsunami wave. The mechanics of a tsunami wave are investigated and the mathematical equations and the physical properties describing a tsunami are developed for the purpose of examining the wave characteristics and studying the change in the physical properties (kinetics) of the wave.

The mathematical and physical properties of a tsunami wave are studied for several representative conditions, and a representative real world tsunami is simulated through computer modeling.

The project develops a model that has the capability of modeling a variety of tsunami conditions; the mathematics and the physics of a tsunami wave are studied through parametric simulations of the behavior of a typical tsunami wave.

The computer program is written using the Excel programming language. The graphs, equations, and charts for a tsunami are studied to understand and create a typical model of a tsunami wave. Together with a working model of a tsunami wave, real time visual representations are outputted using the Excel programming language.

Through the process of mathematical and graphical representation, a tsunami wave is presented in classical mathematical format. All calculations and computer outputs are demonstrated to be correct by hand calculations.

1.0 Introduction

1.1 Purpose

Tsunami is a Japanese word that means, “harbor wave”. The word tsunami is now used to refer to any series of long waves that are generated by a rapid, large-scale disturbance of the sea floor. The purpose of this study is to examine the properties of a tsunami wave and present the mathematical models of the wave based on the dynamics of a tsunami that is applicable to promoting an understanding of the energy of a tsunami.

This report is designed to promote a better understanding of a Tsunami wave. The mechanics of a tsunami wave are investigated and the mathematical equations and the physical properties describing a tsunami are developed for the purpose of examining the wave characteristics and studying the change in the physical properties (kinetics) of the wave. These properties include wave velocity, momentum and total energy Kinetic and potential energies) as the wave approaches and impacts a shoreline. Typically, a tsunami wave strikes a shoreline several hours (3-5 hours) after the initial onset of the tsunami. Due to the localization of tsunamis near a given shoreline, this implies that the superior response near the shoreline is partly due to localized dynamic properties.

Because of the recent seismic activities that have produced large-scale tsunamis that impact a coastal environment, a need for a better understanding of a tsunami wave has arisen. This report develops the total energies and momentum, and shoreline physical characteristics are reviewed for a spectrum of representative tsunami waves and beach conditions.

The mathematical and physical properties of a tsunami wave are studied for several representative conditions, and a representative real world tsunami is simulated through computer modeling.

1.2 Scope

The project develops a model that has the capability of modeling a variety of tsunami conditions; the mathematics and the physics of a tsunami wave are studied through parametric simulations of the behavior of a typical tsunami wave. The kinetics and momentum, based on selected wave velocity and mass are are analyzed and discussed for a range of input values.

1.3 Computer Program

The computer program is written using the Excel programming language. This language can perform calculations and store an inventory of variables in arrays. Information is developed from a variety of sources to gain values for basic equations of state that are assumed to be applicable throughout the entire modeled system. This implies that the situations are expected to occur for the conditions as described. The computer program is capable of selecting specific data values for a typical tsunami, performing the required calculations and presents the results in a chart format. The graphs, equations, and charts for a tsunami are studied to understand and create a typical model of a tsunami wave. All Excel computations are checked by hand calculations for the purpose of verifying the accuracy of outputted results and conclusions. Together with a working model of a tsunami wave, real time visual representations are outputted using the Excel programming language; these aspects are then coupled with the Power Point program to display the output in both hard data format and a time dependent wave motion.

2.0 Project Proposal

2.1 Project Description

This project investigates a typical mid ocean-produced tsunami by conducting parametric studies such as wave velocity, and beach characteristics. This program models wave dynamics and the applicable properties, such as wave velocity, wave momentum, potential energy, and kinetic energy; the dynamic behavior of a representative tsunami wave is modeled in both the open ocean and upon impacting a beach. Obstacles in developing a computational model of a tsunami wave that yields correct inferences for computer wave mechanics are:

- Identification of required input variables
- Technique for selecting applicable parameters
- Understanding both input and output information
- Developing real time visual representation of the mathematical models.

The computer program is designed to describe a typical tsunami wave and identifies restrictions on the development of the model such that the mechanics of the equations produce solutions that describe the physics of a typical tsunami wave impacting on the nearshore (the zone from the shoreline seaward to the line of breakers) and a foreshore (the portion of the shore lying between the normal high an low water marks-the inertial zone) areas.

A technique for enhancing knowledge of the mechanics of a tsunami and a process for evaluating the overall dynamics is the goal of this project. In order to achieve this, a comprehensive set of equations is developed that describe the physics of a tsunami wave, which requires that selective variables be identified that impact the wave dynamics. Parametric equation operations are performed on each individual variable to determine the overall effect of impact each variable has on the comprehensive solution.

3.0 Analytical Methodology

3.1 Mathematical Bases

The Excel program developed to describe the tsunami behavior employs a working mathematical model that describes the physical dynamics of a tsunami wave. These equations describe the mechanics of a wave generated by a large-scale disturbance of the sea and sea floor and the impact of the wave on a shoreline.

The Excel computer program displays a real time presentation applying values calculated using the equations developed in this section to describe the motion of a wave from time of generation until impact on the foreshore of a coast.

The basic equations used to model a tsunami wave are total energy (TE) and momentum (P):

- TE=KE+PE
- KE=½*M*V^2
- PE=M*g*h
- P=M*V

Where:

- KE=kinetic energy of the wave
- PE=potential energy
- P=momentum of the wave,
- M=mass of the water in a specified volume
- V=wave velocity
- g=gravitational constant
- h=height

These equations, as developed, show wave action in relation to real time. The equations to describe wave motion as a function of time and distance are:

$$y(t)=A\sin 2\pi\left(x(t)/\lambda - v/\lambda * t\right)$$

or:

$$y(t)=A\sin 2\pi\left(x(t)/\lambda - t/T\right)$$

where:

- A=amplitude of the wave (assumed to be 0.5 meters (50 cm) in the open ocean)
- λ=wave length
- T=period (seconds)
- $1/T=v/\lambda$
- v=wave velocity
- $y(t)$=vertical position as a function of time
- $x(t)$=horizontal position as a function of time

These variables represent Tsunami wave input parameters.

3.1.1 Conservation of Energy Wave Impacting a Beach

The total energy in a "wave", that is, a mass of water moving at some specified velocity consists of two distinct energy components. These are: kinetic energy and potential energy (refer to Figure 1). The basic equations for the mechanics of wave motion are given in the preceding Section 3.1. The mechanics of a wave are examined for two controlled conditions. First, the assumption is made that no loss of energy occurs, second, loss of energy due to friction at the point of contact between the wave and the ocean floor.

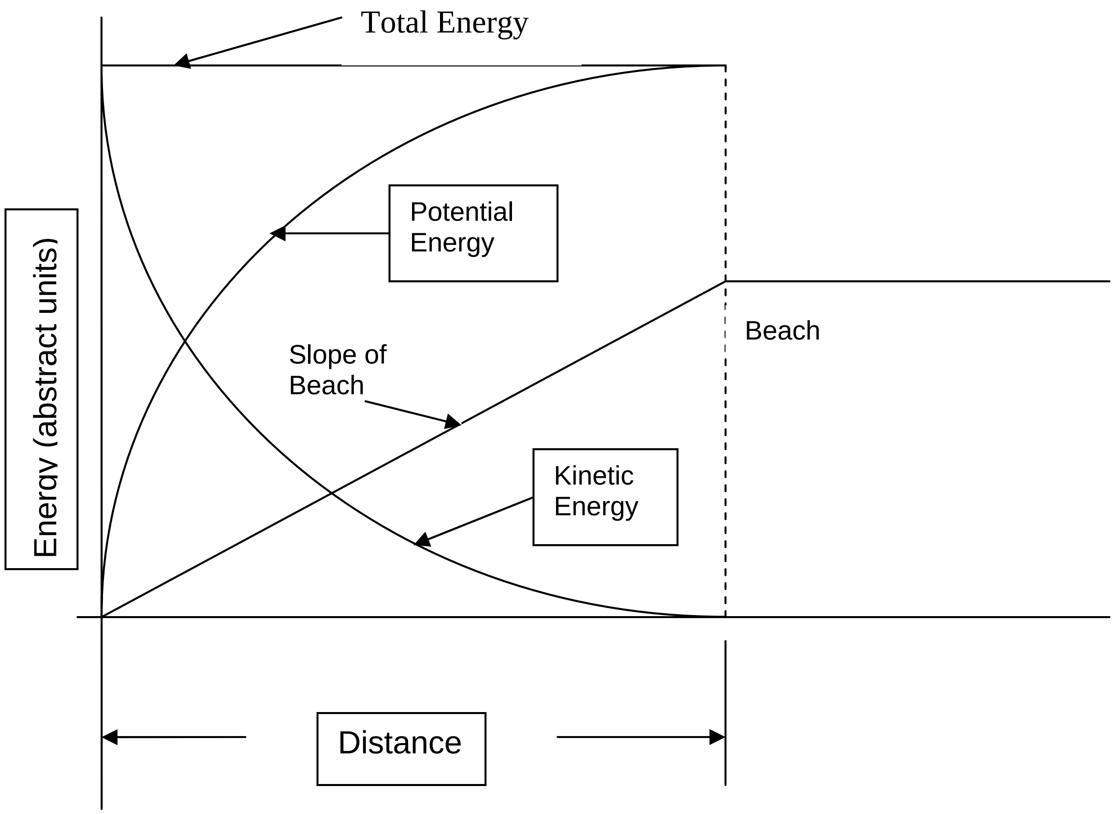


Figure 1 Energy of Wave

Two case studies are analyzed; first, no loss of energy, second, loss of energy. Refer to the appendices for complete analyses of these cases.

3.2 Computer Applications

This computer program is designed to enhance the knowledge of the properties of a tsunami wave. Based on equations for energy and momentum as a function of time, the computer program outputs solutions for the wave mechanics. The computer models utilized in this program are based on two basic inputs. First, a model is developed employing a standard sine function to describe the wave action with no losses of energy. Throughout the analysis, the assumptions are made that the total energy, kinetic energy, and potential energies are conserved. Second, a (future) model is (to be) developed implying that loss of kinetic energy occurs due to friction.

The Excel program is developed using iterated time steps. This method uses two rows of cells, creating a circular reference. The first row of cells saves the values of the function from the last step, and the other row calculates the value of the function at a new time step. This method allows for a specialized program that represents a tsunami wave and allows the wave to be displayed in real time.

The program is developed to find a solution to the wave based on several variables:

- Wave Velocity
- Energy and Momentum
- Mass
- Beach Characteristics- such as slope and depth parameters

All calculations received from the Excel program result in the outputting of the energy and momentum of the wave in open ocean transit and finally as a function of beach characteristics. Specifically the program develops solutions as a function of the beach slope and distance of the foreshore of the coast. Graphs are presented to demonstrate the wave behavior as it impacts and travels up the beach. The Excel program calculates the energy and momentum values for the wave as a function of beach characteristics; the equations can be then modified to employ losses, then the same calculations reiterated and modified incorporating specified dynamic loss functions to demonstrate the resultant wave behavior (refer to Section 6.0). Comparisons can be then made between the two sets of data to represent energy loss and illustrate changes in the waves physical appearance.

4.0 Results

The calculations performed by the computer program accurately represent the properties of a tsunami wave. Calculations also represent the total energies of the wave and respective relationships between kinetic and potential wave energy. Parametric equations are constructed to infer conclusions about certain case conditions. By changing one variable at a time, observations to the different output values can be made. These calculations are based on the assumption that the wave shape follows a standard sine function.

4.1 Calculations

The computer calculations are shown to be correct by comparing calculations performed by the computer programs to matching hand calculations. A significant difficulty encountered in the initial development of the program was the adaptation of the computer model to accept the input variables:

- Wave velocity
- Wave mass
- Friction factor (between point of contact of water and beach)
- Beach characteristics (depth parameters, beach length, and beach slope)

The computer program utilizes selected complex interrelationships between the above noted variables; these final relationships are:

- Wave energy
- Wave momentum
- Change of energy states (kinetic and potential)

The equations are developed to show both energy and momentum for a defined tsunami wave. Refer to the Appendices for complete calculations.

4.2 Charts and Figures

A Microsoft Excel program is used to calculate the respective equations for wave energy and momentum. The output (solutions) as a function of selected input variables are given in Appendices; and special solution for selected energy losses (refer to Section 3.0) are also presented in two dimensional format as a function of time for input conditions selected to demonstrate the versatility of the program.

Figures corresponding to these input/output conditions are created using Microsoft Excel. The equations describing the time dependent motion of the wave (refer to section 3.0) are formatted into the Microsoft Excel programming language to show wave motion as a function of time and demonstrate the versatility of the program in displaying unique functions.

The figures present several unique conditions (selected especially for this study); these figures model a representative set of tsunami conditions that demonstrate the capabilities of this program to show “real time” motion of the wave.

5.0 Conclusions

5.1 Mathematical Models

Through the process of mathematical and graphical representation, a tsunami wave is presented in classical mathematical format. All calculations and computer outputs are demonstrated to be correct by hand calculations.

5.2 Computer Program

Calculating the mechanics of the wave requires basic selected inputs, value parameters characterized as equations of motion in a series of specific equations. The computer program initially prompts the user to input easily referenced variables from the reference text and journal articles. Using these basic variables to specify the unique characterization of the tsunami, the program selects values from the referenced documents in order to provide the equations with accurate information on how the wave reacts under the specified input conditions. These are known as the dynamics of the wave. Combining these equations and the variable input parameters defined by the user, Excel calculations are performed to examine wave dynamics for beach characteristics and wave dynamics (refer to Appendices for complete set of calculations).

5.3 Results

The program employs the basic dynamics of wave action to develop solutions for wave characteristics, in this case a tsunami, as the wave impacts on a specified shore. Wave action is specified for selected input conditions and resultant output values are calculated. Purpose of these calculations is to demonstrate that a set of output can be correctly determined for a specified set of input conditions (in this case, a defined tsunami wave), and the calculated values for energy and momentum can be employed to characterize wave behavior.

5.4 Word Processor Program

The written portion of the program is created using Microsoft Word; the calculations are performed using a modified Excel program for the purpose of displaying the tsunami wave in real time motion.

6.0 Recommendations for Future Work

To improve the project beyond what has been accomplished to date, it is recommended that improvements be made on the cross section view of a specified tsunami wave for a larger wave section. The program should also be written to represent a plan view of the wave and a specified coastline. Loss of energy and changes in wave characteristics due to reflection, refraction and energy loss, and deflection should also be added to represent more realistic behavior.

7.0 References

1. Gonzàlez, F.I., K. Satake, E.F. Boss, H. O. Mofjeld. Edge Wave and Non-Trapped Modes of the 25 April 1992 Cape Mendocino Tsunami.
2. Mofjeld, H.O., V.V. Titov, F.I. Gonzàlez, J.C. Newman. Analytic Theory of Tsunami Wave Scattering in the Open Ocean with Application to the North Pacific. NOAA Technical Memorandum OAR PMEL-116. January 2000.
3. Thurman, Harold V. Introductory Oceanography. Columbus: Bell & Howell, 1975.

8.0 Appendices

Appendix 1

Equations to Describe Wave Motions as a Function of Time

These equations describe wave motion as a function of time. The following equation describes the basic for of wave motions.

$$y(t)=A\sin2\pi(x(t)/\lambda-v/\lambda*t)$$

or:

$$y(t)=A\sin2\pi(x(t)/\lambda-t/T)$$

For tsunami wave input parameter:

- = wave length = 400 m = 40,000 cm.
- T = period (seconds)
- I/t = v/λ = 1670/40000 = 0.042 sec^{-1}
- V = wave velocity = 600 km/hr = 600000/3600 = 16700 cm/sec
- Y(t) = vertical position as a function of time (cm)
- X(t) = horizontal position as a function of time (cm)

Substituting these values: into the equation for y(t), the vertical displacement of the wave as a function of horizontal distance is:

$$Y(t) = A \sin 2\pi\ (x(t)/40000 – 16700/40000 * T)$$
$$= A \sin 2\pi\ (x(t)/40000\ 0.42\ T)$$

The amplitude of the wave is given by A and for this example the wave height (amplitude) is assumed to b 0.40 meters or 40 cm.

$$Y(t) = 40\ 2\pi\ (x(t)/40000\ 0.42\ T)$$
$$= 40\ 2\pi\ (1.57 * 10^{-4} x(t) – 2.62T)$$

This equation describes the vertical displacement (or motion) of the wave as a function of time and downrange position – x(t).

As example, assumes the a following value, for x(t) and t:

- X(t)= 10000 cm.
- T(t)= 100sec.

The vertical displacement of the wave is.

$$Y(t) = 40 \text{ Sin } [\ 1.57^{-4}\ * (10000) – 2.62\ (100)]$$
$$= 40 \text{ Sin } [1.57\ \ - 262.0]$$
$$=(-260.4)$$

Where the argument of the sine function is in <u>radians, </u>solving for y(t).

$$Y(t) = [40\ 0.317]$$
$$=12.67 \text{ cm}$$

In other words, at the specified time (100 seconds) and downrange distance (10000cm) the height of the wave is 12.67cm for the assumed input conditions.

The vertical velocity of the wave can be determined by differentiating the basic equations for y(t).

$$y(t) = 40 \sin [1.57 \times 10^{-4} (x(t)) –2.62t]$$
$$dy(t)/dt = \text{vertical velocity} =Vv(t)$$
$$=d \{40 \sin [1.57 \times 10^{-4} (x(t)) –2.62t]\}/ dt$$

The variable to be differentiated is 2.62t since that is the term containing time as a variable, thus:

$$dy\ (t)/dt = Vv\ (t)$$

$$= 40 \{-2.62\{\cos[(1.57 \times 10^{-4} (10000) - 2.62\ t(t)]\}\}$$

$$= -104.80\{\cos[1.57 \times 10^{-4}\ x\ (t) - 2.62\ t\ (t)]\}$$

Substituting the ordinal assumed value: for x (t) and t(t) [x(t) = 10000cm and t(t) = 100sec] the vertical or transverse wave velocity is

$$Vv(t) = -104.80\ \{\cos[1.57 \times 10^{-4} (10000) - 2.62\ (100)]\}$$

$$= -104.80 \cos\ [\ 1.57 - 2.62]$$

$$= -104.80 \cos (-260.43)$$

$$= -104.80\ (-0.948)$$

= 99.4 cm/sec (upward w/repeat to horizontal plane)

[Note : A minus sign would indicate a downward velocity]

Consequently, for the assumed input conditions, at a downrange distance of x(t) = 10000cm, time t(t) = 100sec, the following conditions of wave are:

y(t) = 12.67 cm

x(t) = 10000 cm

v(t) = 16700 cm/sec

vv(t) = 99.4 cm/sec

Appendix 2

Conservation of Energy

Wave Impacting a Beach

The total energy in a “wave”, that is mass of water moving at some specified velocity consists of two distinct energy components. There are kinetic energy and potential energy. The mechanics of a wave is presented below for the condition of no energy losses [a follow-on study is planned to examine energy loss due to friction between the wave (or mass of water) and the ocean floor (as it rises to the beach level)]. A plot of these two terms is represented in Figure 1.

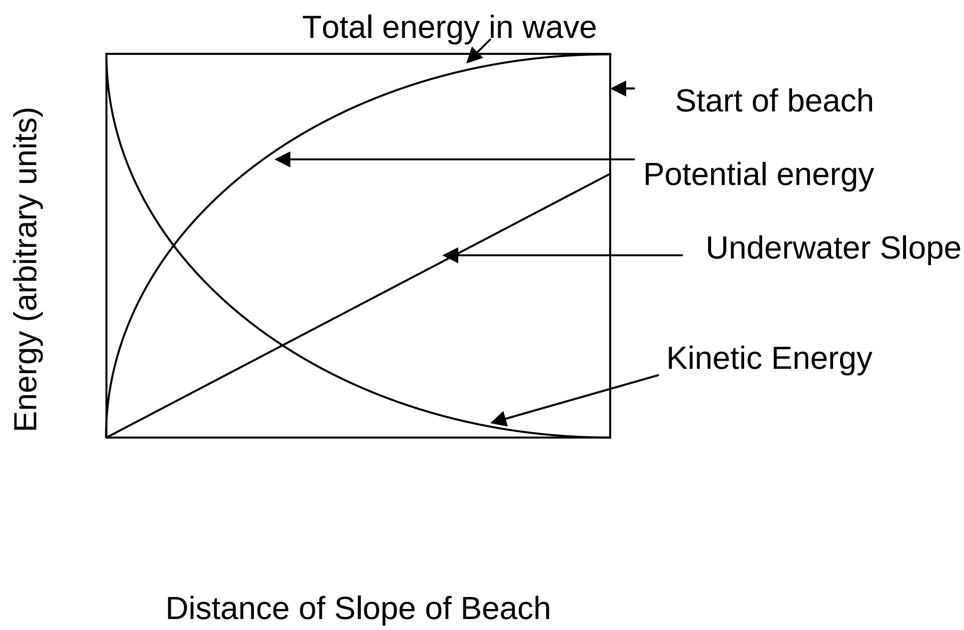
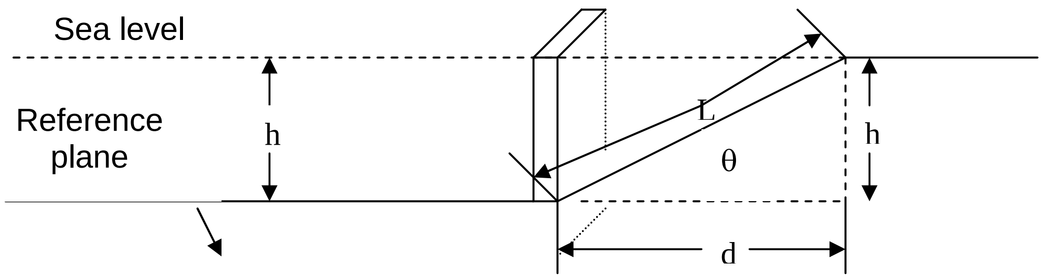


Figure 1- Energy of Wave

As illustrated in Figure 1 (which is a hypothetical representation of the energy distribution), as the wave enters the zone of the below-surface region (0 – x), the kinetic energy and the potential energy change in magnitude but for the no-loss-of-energy condition the total energy remains constant. This is analyzed in the following section .

No Loss of Energy

A slug of water is assumed to encounter the bottom or leading edge of the submerged beach as shown in Figure 2.



The mass of water in the elemental volume, ΔV, is given by:

$$\text{Mass of water} = m_w = \rho_{sw} * \text{vol. of sample slug}$$
$$= \rho_{sw} * h\, t\, w$$

Where w is the width of the slug of water but for purposes of this analysis it is assumed to be of unit length (that is,1) so that the mass of water can be re-written as:

$$m_w = \rho_{sw} * h\, t$$

The kinetic energy of the slug of water is given by:

$$KE_{slug} = ½\, m * v^2$$

Where v = the velocity of the slug of water as is travels up the slope.

The potential energy of the slug of water is given by:

$$PE = mg\, H$$

Where H is the height of the center of mass of the slug above the reference plane. Since the water mass is assumed uniform and dimensionally constant, the potential energy can be written in terms of total sub – surface height h as:

$$PE = \frac{1}{2} mgh$$

As previously noted, the total energy of the wave is assumed too remain constant (for care 1); hence

Total Energy (TE) = Kinetic Energy (KE) + potential energy (PE)

Or $$TE = KE + PE$$

And substituting for these terms:

$$TE = \frac{1}{2} mv^2 + \frac{1}{2} mgh$$

Next, the solution must account for the underwater slope of the beach, or angle θ. Introducing the change in height of the slug of water as it traverses up the underwater slope, the term for h is modified as follows:

$$H = L \sin \theta$$

This term allows the equation to account for the length or "run" of the underwater distance the slug of water travels as is rises to the top of the beach. Also, the down-range distance d, which is the horizontal distance the wave travels as the wave travels up the slope of the beach is given by:

$$d = L \cos \theta$$

or using another expression

$$d = h / \tan \theta$$

As example, for a fixed height of h (the distance the surface of the ocean floor is below the surface of the water) and an assumed slope of 30^{o} for the underwater rise of the ocean floor the distance d is:

$$D = 300/\tan 10^{o}$$
$$= 1701.4 \text{ meters}$$

for an initial underwater height at the start of the “beach” rise of 300 meters. Continuing with the trigonometric relationships, the distance L for this example is:

$$L = d/\cos\theta$$
$$=1701.4 / \cos 10$$
$$= 1727.6 \text{ meters}$$

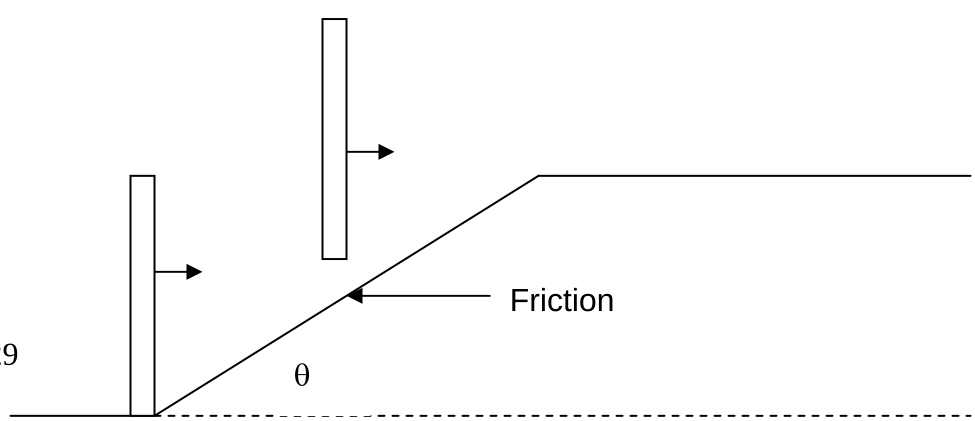
Therefore, for a height of 300 meters and an angle of 10^{o}; the length of underwater beach run is 1727.6 meters.

A complete set of distances and underwater lengths can be evaluated as a function of the basic input variables slope of beach (θ) and height (h) of underwater beach.

The foregoing equations do not include a loss factor for the friction beween the water and beach surface. This is evaluated in the next section.

Loss of Energy (future study)

The energy loss assumed for this calculation is the friction between the slug of water and the (underwater) surface of the beach as shown in the figure.



Appendix 3

Review of the Total Energy In the Wave

This paper examines the total energy of the wave as it impacts the (underwater) shore and moves up the beach. Recall from Appendix 1 that the total energy of the wave is the summation of the kinetic energy and potential energy.

$$\text{Total Energy} = KE + PE$$

Recall Figure 2 (Appendix 1), at the instant the wave enters the zone of the beach, the total energy is equal to the kinetic energy. As the wave moves up the beach slope, the total energy is the summation of the kinetic energy plus the potential energy. From Appendix 1, the kinetic energy in the wave is:

$$KE = -\tfrac{1}{2}\, mv_o^2 + \tfrac{1}{2}\, mv_f^2$$

where v_o = velocity of wave at start of slope

v_f = velocity of wave at top of slope

The work performed by the mass of water as it moves up the beach is given by:

$$W = -mg \sin\theta \times d - \mu\, mg \cos\theta \times d$$

Where θ = slope of the beach

d = distance water moves up the beach

μ = friction factor between water and beach

g = gravity = 9.8 m/sec ^{2}

Equating change in Kinetic Energy to the change in potential energy of the mass of water (m) as the water moves up the beach:

$$-\tfrac{1}{2}\, mv_o^2 + \tfrac{1}{2}\, mv_f^2 = -mg \sin\theta \times d - \mu\, mg \cos\theta \times d$$

The mass (m) cancels in the equation, and the equation can be re-written as:

$$-v_o^2 + v_f^2 = 2\,(-g \sin\theta \times d - \mu\, g \cos\theta \times d)$$

The velocity v_f at the top of the beach is:

$$v_f^2 = v_o^2 - 2gd\ (\sin\theta + \mu\cos\theta)$$

Divide by cos θ and rearranging terms:

$$v_f^2 = v_o^2 - 2gd\cos\theta\ (\mu + \tan\theta)$$

or

$$v_f = [v_o^2 - 2gd\cos\theta\ (\mu + \tan\theta)]^{1/2}$$

This equation describes the velocity of the wave as the wave moves up the underwater beach of slope "θ". If, as example, it is assumed that there is no friction, that is μ = 0, the above equation reduces to:

$$V_f = [v_o^2 - 2gd\sin\theta]^{1/2}$$

As shown in Appendix 1 for an assumed set of input conditions, namely:

$$D = 300/\sin\theta = 300/\sin 10^o = 1727.6\text{ m.}$$

$$\theta = 10^*$$

$$\mu = 0.01$$

$$V_f = [(166.67)^2 - 2(9.8)(1727.6)(\cos 10^o)(0.01 + \tan 10^o)]^{1/2}$$
$$= 146.9\text{ m/sec.}$$

This is the velocity or the water (or wave front) as it reaches the end of the underwater beach.

The change in Kinetic Energy of the wave is given by :

$$
\begin{aligned}
KE &= KE \text{ @ start} - KE \text{ @ top} \\
&= \tfrac{1}{2}\, mv_o^2 - \tfrac{1}{2}\, mv_i^2 \\
&= \tfrac{1}{2}\, m\, [(166.7)^2 - (146.9)^2] \\
&= m \times 3106.7 \text{ Joules}
\end{aligned}
$$

This represent the LOSS in Kinetic Energy as the wave proceeds up the underwater beach. This value also represents the change in potential energy which can be equated to the change in Kinetic Energy. The change in potential energy is given by

$$
\begin{aligned}
PE &= mgd \sin\theta + \mu\, mgd \cos\theta \\
&= m(9.8)(1727.6)[\sin 10^o + 0.01 \cos 10^o] \\
&= m \times 3106.7 \text{ Joules}
\end{aligned}
$$

This basically agrees with the KE term above.

If there is no friction factor assumed then the change in the Potential Energy (PE) in given by:

$$PE = mgd \sin\theta$$

And the final velocity at the top of the underwater beach is given by:

$$- v_o^2 + v_f^2 = - 2dg \sin\theta$$

or:

$$
\begin{aligned}
v_f^2 &= v_o^2 - 2dg \sin\theta \\
&= [(166.67)^2 - 2\,(9.8)(1727.6) \sin 10^o]
\end{aligned}
$$

v_f = 148.0 m/sec.

As shown, the “assumed” friction factor for water does not significantly affect the velocity of the wave. The principle “changes in energy” are due to the change in potential. Considering a strictly case where energy is conserved, that is, no loss due to friction, the change in potential energy is given by (as noted above):

$$PE = mgd \sin\theta$$
$$= m \times (9.8)(1727.6) \sin 10^o$$
$$= m \times 2940.0 \text{ Joules}$$

The change in Kinetic Energy for these conditions is given by:

$$KE = ½ mv_o^2 – ½ mv_f^2$$

Where $$v_f^2 = v_o^2 – 2 \text{ gd} \sin \theta$$
$$= (166.7)^2 – 2 (9.8)(1727.6) \sin10^o$$
$$= 21899.0$$

or $$v_f = 148.0 \text{ m/sec.}$$

Hence the change in Kinetic Energy is given by:

$$KE = ½ m v_o^2 – ½ m v_f^2$$
$$= ½ m [5884.9]$$
$$= m \times 2942.4 \text{ Joules}$$

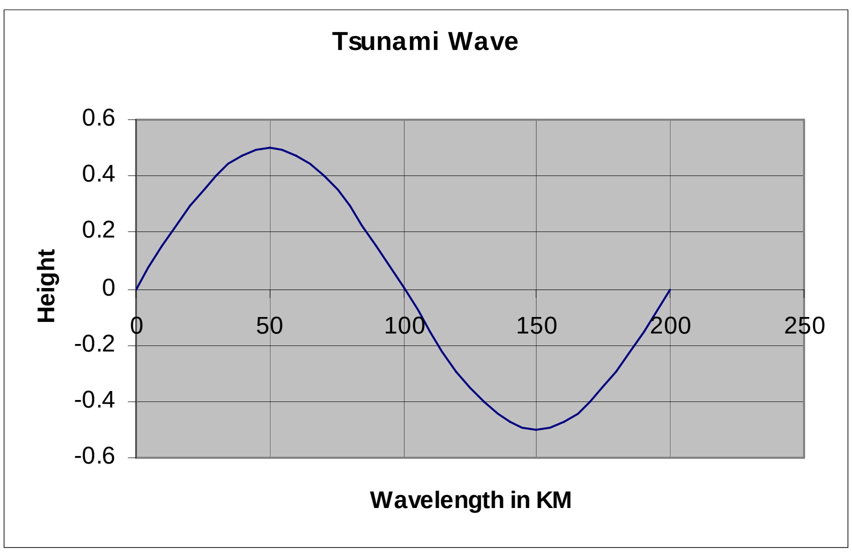
and this compares with the PE for no energy losses on page TE – 4 (Note: The mass term is left in the equation and all terms can be normalized to the mass by dividing through by m.)

Appendix 4

Excel Calculations

Sine Function in Open Ocean

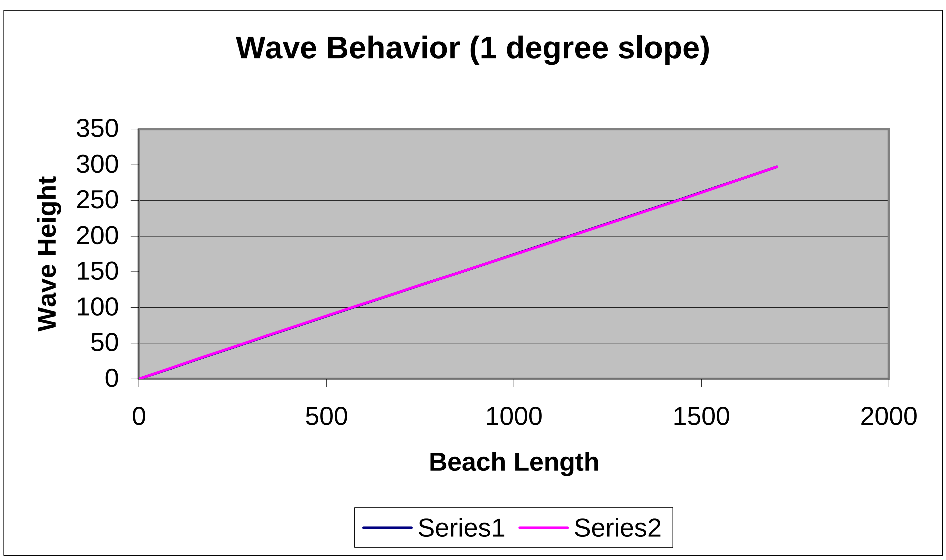
0	0
0.154508	10
0.293892	20
0.404508	30
0.475528	40
0.5	50
0.475529	60
0.404509	70
0.293893	80
0.15451	90
1.33E-06	100
-0.15451	110
-0.29389	120
-0.40451	130
-0.47553	140
-0.5	150
-0.47553	160
-0.40451	170
-0.29389	180
-0.15451	190
-2.7E-06	200



Wave Behavior (Impacting Beach)

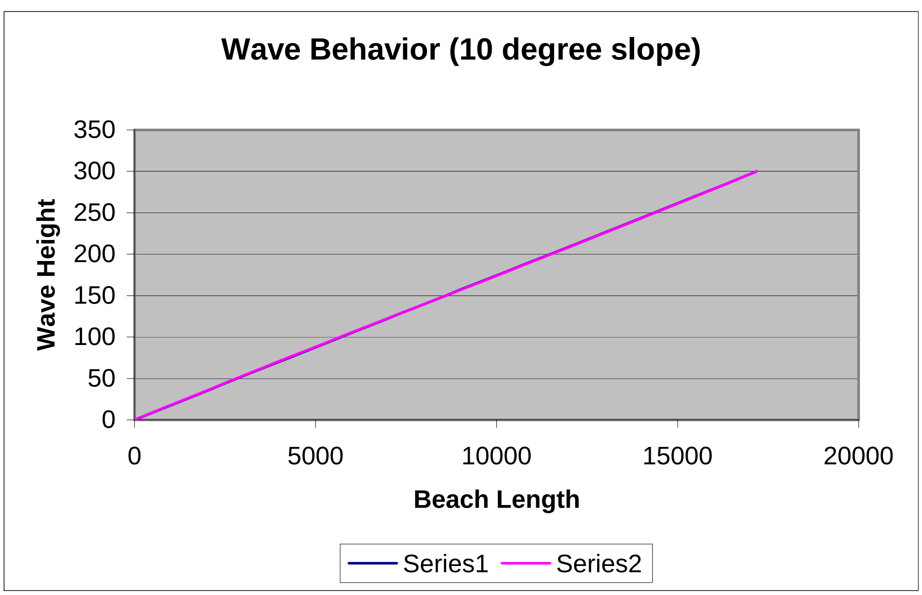
1^0 Beach Slope

0.174533		
0	0	0
85.06923	15.00188	15.15639
170.1385	29.98863	30.28252
255.2077	44.94661	45.35112
340.2769	59.865	60.34053
425.3461	74.73684	75.23684
510.4154	89.55974	90.03527
595.4846	104.3361	104.7406
680.5538	119.0728	119.3667
765.623	133.7808	133.9353
850.6923	148.4737	148.4737
935.7615	163.1665	163.012
1020.831	177.8745	177.5806
1105.9	192.6113	192.2068
1190.969	207.3876	206.9121
1276.038	222.2105	221.7105
1361.108	237.0824	236.6068
1446.177	252.0008	251.5962
1531.246	266.9587	266.6648
1616.315	281.9455	281.791
1701.385	296.9474	296.9474



10^0 Beach Slope

0.017453		
0	0	0
859.451	15.15474541	15.30925
1718.902	30.29436649	30.58826
2578.353	45.40521938	45.80973
3437.804	60.47647625	60.952
4297.255	75.5011852	76.00119
5156.706	90.47695074	90.95248
6016.157	105.4061683	105.8107
6875.608	120.2957898	120.5897
7735.059	135.156643	135.3112
8594.51	150.0023717	150.0024
9453.961	164.8481003	164.6936
10313.41	179.7089531	179.4151
11172.86	194.598574	194.1941
12032.31	209.5277909	209.0523
12891.77	224.5035556	224.0036
13751.22	239.5282637	239.0527
14610.67	254.5995198	254.195
15470.12	269.7103722	269.4165
16329.57	284.8499929	284.6955
17189.02	300.0047381	300.0047



Wave Momentum and Kinetic Energy Calculations

Velocity Km/hr	velocity m/sc	KE in MKS	KE in Enginering	Mass in MKS	Velocity in ft/sc
600	166.67	4.26E+10	1.01E+12	3066000	546.92
620	172.22	4.55E+10	1.08E+12	3066000	565.15
640	177.78	4.85E+10	1.15E+12	3066000	583.39
660	183.33	5.15E+10	1.22E+12	3066000	601.62
680	188.89	5.47E+10	1.30E+12	3066000	619.85
700	194.44	5.80E+10	1.37E+12	3066000	638.08
720	200.00	6.13E+10	1.45E+12	3066000	656.31
740	205.56	6.48E+10	1.53E+12	3066000	674.54
760	211.11	6.83E+10	1.62E+12	3066000	692.77
780	216.67	7.20E+10	1.70E+12	3066000	711.00
800	222.22	7.57E+10	1.79E+12	3066000	729.23
820	227.78	7.95E+10	1.88E+12	3066000	747.46
840	233.33	8.35E+10	1.98E+12	3066000	765.69
860	238.89	8.75E+10	2.07E+12	3066000	783.92
880	244.44	9.16E+10	2.17E+12	3066000	802.15
900	250.00	9.58E+10	2.27E+12	3066000	820.39

velocity m/sc	Momentum in MKS
166.67	5.11E+08
172.22	5.28E+08
177.78	5.45E+08
183.33	5.62E+08
188.89	5.79E+08
194.44	5.96E+08
200.00	6.13E+08
205.56	6.30E+08
211.11	6.47E+08
216.67	6.64E+08
222.22	6.81E+08
227.78	6.98E+08
233.33	7.15E+08
238.89	7.32E+08
244.44	7.49E+08
250.00	7.67E+08

Appendix 5

Excel Program

Typical Sine Function

		Time	1.00
Init Flag	TRUE		
1			
-1.00			

0	-0.0975451
10	0.0587686
20	0.2093297
30	0.3394001
40	0.4362478
50	0.4903925
60	0.4965343
70	0.4540719
80	0.3671619
90	0.2443116
100	0.0975464
110	-0.0587673
120	-0.2093285
130	-0.3393992
140	-0.4362471
150	-0.4903923
160	-0.4965345
170	-0.4540725
180	-0.3671628
190	-0.2443127
200	-0.0975477