Whirlpools, not just another flush of the toilet.

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Have you ever flushed your toilet and happened to notice that it swirls around in a counterclockwise circular motion? That circular motion creates an "eye" in the center of the toilet, forming a whirlpool, also known in science as a vortex. Our project is going to demonstrate that just as the toilet can create a vortex, we can simulate the factors for how this occurs. We will demonstrate how an agent-based program (NetLogo) can create a math-based model of a vortex.

The Problem

This project will demonstrate how using simple rules and cellular automata we can create a vortex. The way were going to use cellular automata is to define how we can use simple rules to recreate a math base problem. The Definition of Cellular Automata is a dynamical system which is a discrete in space and time operate on a uniform regular lattice and are characterized by "local" interactions

Definition of a Vortex

A vortex is a swirling body or circular motion that tends to form a cavity or vacuum. When the cavity forms it apparently pulls the fluid into the cavity. Our project is attempting to demonstrate the simple factors needed to form a vortex on land or in water, whether it is in the middle of the ocean, inside your bathroom, out on the plains (tornado), or on the ocean (hurricane/typhoon). We currently believe vortices are formed when an element (being air or water) is moving in a circular motion and the center has so much pressure on it that it caves in, creating the pull/suction that vortices are so well known for. A vortex in the toilet works on the same principles … when you flush the water gets pored in until the water-trap keeping the water there can't hold the pressure and releases forming the cavity.

How some Vortices in Nature are made?

There are two popular vortices in nature that everybody knows about. The first one or the most famous is the tornado. These are conman in the middle part of America in states like Missouri, Nebraska, Kansas, etc. These are formed when two air masses clash together. The air masses need to have a high pressure mass and a low pressure mass. When these collide they create a massive thunderstorm and when they become a massive thunderstorm, eye of the storm may be forced down to create a tornado. The reason for why it dissipates could be the loss of strength in the storm. The other popular one is a whirlpool. This forms when the tides in the water collide and start to make a downdraft.

Definition of a whirlpool.

"A whirlpool is a swirling body of water usually produced by ocean tides."(Wikipedia) Whirlpools are usually described as a vortex that is what they are but they are mainly just a small subset of free vortices. A free vortex is when the tangential velocity increases and the angular momentum decreases. Whirlpools can be seen when a bath or sink is daring but they are formed in a totally different way then those in nature are produced. Here is a picture of what we are trying to recreate

The Formulas.

Fluid Dynamics

Fluid dynamics is the study of the flow of fluids such as water. We did research on fluid dynamics to know how water would react. This is one of the most important details to our project, because if we did not know the way fluid reacts in situations then this project would never work. The definition of fluid dynamics is “the scientific study of the property, properties and behavior of moving fluids.”(ALLWORDS.com) Yes this info is important to are project but we are not using these formulas to make our model.

Bernoulli’s Principle

Bernoulli’s principle is the study fluid and pressure and the way that they interact with each other. Understanding this principle is important and the reason that it is important is to make a whirlpools or tornado you need pressure. The understanding of how liquid and pressure react helps us know that if we add more pressure then the whirlpool could just collapse, go faster or slow the whirlpool down. The principle states that “as the speed of the fluid increases the pressure from the fluid decreases."(Wikipedia) The slower the speed of the fluid is the more powerful the pressure gets. So if we had lots of pressure then the whirlpool would slow and eventually collapse and if we had a little bit of pressure then the whirlpool would be fast. "The well-known Bernoulli's equation can be derived by integrating Euler's equation along a streamline, under the assumption of constant density and a sufficiently stiff equation of state." (Wikipedia) A really easy way to see this in action is by taking a hair dryer and turning it on and put a ping pong ball above the air. This shows that when you turn the air up the ball moves higher and faster and when you turn the air low the ball gets slow and falls the air equals the pressure and ping pong ball equals flow.

Euler equations

Euler equations are about inviscid flow and they are based on Navier-Stokes equations with zero viscosity and heat conduction terms. These are some of the formulas that helped us learn about whirlpools and will keeping helping us learn. This is

Euler equations in differential form

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0$$

$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\mathbf{u} \otimes (\rho \mathbf{u})) + \nabla p = 0$$

$$\frac{\partial E}{\partial t} + \nabla \cdot (\mathbf{u}(E + p)) = 0,$$

"Where ρ is the fluid mass density, u is the fluid velocity vector, with components u, v, and w, $E = \rho e + \frac{1}{2} \rho (u^2 + v^2 + w^2)$ is the total energy per unit volume, with e is the internal energy per unit mass for the fluid, and p is the pressure." (Wikipedia) The second equation is

$$\frac{\partial (\rho u_j)}{\partial t} + \sum_{i=1}^{3} \frac{\partial (\rho u_i u_j)}{\partial x_i} + \frac{\partial p}{\partial x_j} = 0,$$

where the i and j subscripts label the three Cartesian components: $(x_1 , x_2 , x_3) = (x , y , z)$ and $(u_1 , u_2 , u_3) = (u , v , w)$. The above equations are presented in conservation form the next form of Euler equations they are in vector form.

$$\frac{\partial \mathbf{m}}{\partial t} + \frac{\partial \mathbf{f}_x}{\partial x} + \frac{\partial \mathbf{f}_y}{\partial y} + \frac{\partial \mathbf{f}_z}{\partial z} = 0,$$

where

$$\mathbf{m} = \begin{pmatrix} \rho \\ \rho u \\ \rho v \\ \rho w \\ E \end{pmatrix} \qquad \mathbf{f}_x = \begin{pmatrix} \rho u \\ p + \rho u^2 \\ \rho u v \\ \rho u w \\ u(E+p) \end{pmatrix} \qquad \mathbf{f}_y = \begin{pmatrix} \rho v \\ \rho u v \\ p + \rho v^2 \\ \rho v w \\ v(E+p) \end{pmatrix} \qquad \mathbf{f}_z = \begin{pmatrix} \rho w \\ \rho u w \\ \rho v w \\ p + \rho w^2 \\ w(E+p) \end{pmatrix}.$$

e

This form makes it clear that f_x, f_y and f_z are fluxes."(Wikipedia) theses are his equations in non-conversation form with flux Jacobians.

$$\frac{\partial \mathbf{m}}{\partial t} + \mathbf{A}_x \frac{\partial \mathbf{m}}{\partial x} + \mathbf{A}_y \frac{\partial \mathbf{m}}{\partial y} + \mathbf{A}_z \frac{\partial \mathbf{m}}{\partial z} = 0.$$

where A_x, A_y and A_z are called the flux Jacobians, which are matrices equal to:

$$\mathbf{A}_x = \frac{\partial \mathbf{f}_x(\mathbf{s})}{\partial \mathbf{s}}, \qquad \mathbf{A}_y = \frac{\partial \mathbf{f}_y(\mathbf{s})}{\partial \mathbf{s}} \qquad \text{and} \qquad \mathbf{A}_z = \frac{\partial \mathbf{f}_z(\mathbf{s})}{\partial \mathbf{s}}.$$

(Wikipedia)

These equations are non liner just like the original Euler equations the only different in this form where the state vector m varies lightly. Now here are his equations in Linearized form. “The linearized Euler equations are obtained by linearization of the Euler equations in non-conservation form with flux Jacobians, around a state m = m_0, and are given by (Wikipedia):

$$\frac{\partial \mathbf{m}}{\partial t} + \mathbf{A}_{x,0} \frac{\partial \mathbf{m}}{\partial x} + \mathbf{A}_{y,0} \frac{\partial \mathbf{m}}{\partial y} + \mathbf{A}_{z,0} \frac{\partial \mathbf{m}}{\partial z} = 0,$$

where $A_{x,0}$, $A_{y,0}$ and $A_{z,0}$ are the values of respectively A_x, A_y and A_z at some reference state m = m_0.(Wikipedia) The last case of Euler equations are when Transformation to uncoupled wave equations for the one-dimensional case.

$$\frac{\partial \mathbf{m}}{\partial t} + \mathbf{A}_{x,0} \frac{\partial \mathbf{m}}{\partial x} = 0.$$

The matrix $A_{x,0}$ is diagonalizable, which means it can be decomposed into:

$$\mathbf{A}_{x,0} = \mathbf{P}\boldsymbol{\Lambda}\mathbf{P}^{-1},$$

$$\mathbf{P} = [\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3] = \begin{bmatrix} 1 & 1 & 1 \\ u-a & u & u+a \\ H-ua & \frac{1}{2}u^2 & H+ua \end{bmatrix},$$

$$\boldsymbol{\Lambda} = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} = \begin{bmatrix} u-a & 0 & 0 \\ 0 & u & 0 \\ 0 & 0 & u+a \end{bmatrix}.$$

Here r_1, r_2, r_3 are the right eigenvectors of the matrix $A_{x,0}$ corresponding with the eigenvalues λ_1, λ_2 and λ_3.

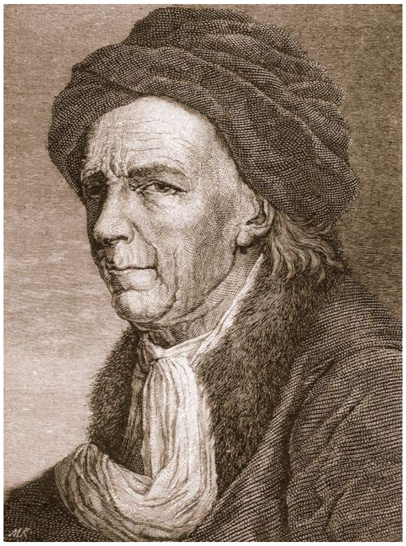
Defining the characteristic variables as:

$$\mathbf{w} = \mathbf{P}^{-1}\mathbf{m},$$

Since $A_{x,0}$ is constant, multiplying the original 1-D equation in flux-Jacobian form with P^{-1} yields:

$$\frac{\partial \mathbf{w}}{\partial t} + \mathbf{\Lambda}\frac{\partial \mathbf{w}}{\partial x} = 0$$ (Wikipedia)

The above formulas show how complex whirlpools are. This is tradition science and very difficult to difficult to model because all mathematical formulas are estimations



Euler, Leonard

The Model

The program we used to model our whirlpool was net logo. Net logo is an agent based modeling program. The way we are using this is by shooting water thru little openings on the side then we have dead zones setup in parts of the model so the water already in the world can move. We have the openings setup on all four corners so that we can get a circular motion. Net logo is best used to show individuals and not really what we are doing but we have made it work. The reason that we chose net logo is that it is the program that we know the best. We also decide to use net logo because we can't put in a simple set of rules and get a math based problem out for example are project a whirlpool uses a lot of math but where using simple gas laws to try and produce a whirlpool. Here are the ideal gas laws presented in flux Jacobian form.

The x-direction flux Jacobian:

$$\mathbf{A}_x = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ \hat{\gamma}H - u^2 - a^2 & (3-\gamma)u & -\hat{\gamma}v & -\hat{\gamma}w & \hat{\gamma} \\ -uv & v & u & 0 & 0 \\ -uw & w & 0 & u & 0 \\ u[(\gamma-2)H - a^2] & H - \hat{\gamma}u^2 & -\hat{\gamma}uv & -\hat{\gamma}uw & \gamma u \end{bmatrix}.$$

The y-direction flux Jacobian:

$$\mathbf{A}_y = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ -vu & v & u & 0 & 0 \\ \hat{\gamma}H - v^2 - a^2 & -\hat{\gamma}u & (3-\gamma)v & -\hat{\gamma}w & \hat{\gamma} \\ -vw & 0 & w & v & 0 \\ v[(\gamma-2)H - a^2] & -\hat{\gamma}uv & H - \hat{\gamma}v^2 & -\hat{\gamma}vw & \gamma v \end{bmatrix}.$$

The z-direction flux Jacobian:

$$\mathbf{A}_z = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ -uw & w & 0 & u & 0 \\ -vw & 0 & w & v & 0 \\ \hat{\gamma}H - w^2 - a^2 & -\hat{\gamma}u & -\hat{\gamma}v & (3-\gamma)w & \hat{\gamma} \\ w[(\gamma-2)H - a^2] & -\hat{\gamma}uw & -\hat{\gamma}vw & H - \hat{\gamma}w^2 & \gamma w \end{bmatrix}.$$

Where $\hat{\gamma} = \gamma - 1$.

The easy form of the ideal gas law that most people know is

$$pV = nRT$$

where

p is the absolute pressure of the gas, *V* is the volume of the gas,

n is the number of moles of gas,

R is the universal gas constant,

T is the absolute temperature.

The value of the ideal gas constant

We are using Ideal gas law in our model .

Results

The results that we got were a lot of little whirlpools in different sections. These little ones would then grow out until they ran into other whirlpools. Once they ran into other whirlpools in all of their sides they would stop growing and they would stay in the same pattern forever. This is something that we did not want to happen. What we wanted to happen is that these little whirlpools to run into each other and they would combine to form one huge whirlpool. One thing that we found interesting about this project is that some whirlpools grew faster than others. We also never had any whirlpools start out in the middle of the area. The whirlpools usually were formed in the corners or on the sides of the screen. These are the results of are first model the one we are working on now is using the gas laws to make a whirlpool form.

What we did this year.

This year two team members went to Glorieta. We met with Nick Bennett several times throughout the school year. We would have team meetings every Saturday from ten to twelve in the morning or longer. We would meet up at night library and get help from the math teacher. In the mornings before school we would work with some of the science teachers for help with the physics part of the problem. We would later travel to New Mexico Tech to see their engineering library. The next thing we would do is travel up to Albuquerque to take the tour of Sandia. When on that trip we split into two groups and half went to the unm tour and the other going to the sandia tour. When at the hotel we got help once again from nick Bennett on our code. He would then come down the following weekend to help the other two teams and also our team. While down in Artesia he helped us put the final touches on our project and also helped us with a little of the physics that we could not figure out.

How this year was

With a project such as this one we have learned that a whirlpool, which looks very simple, is very difficult to discover what causes it to form. We hope that our project shows just how interesting vortices are in their formation, so next times you are flushing your toilet just think… how does it happen?

What we plan on doing next

If we can are next steps we will take with the project will be that we might add objects into the whirlpool like a ball or a small boat. If we have the time we where thinking of sending a small wave at first and then gradually increasing in size of the wave to almost tide wave size. The problem we are solving with that would be if the whirlpool stopped the wave could of made a man made whirlpool and saved lives when the big tide wave hit the over seas and wiped out a city. If we can model this problem we could save millions of lives. That is what we going to do to take this project farther.

Appendix 1 : Sudo Code

2 : Code

3:

4:

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