

EXCELLANTS

NEW MEXICO
SUPERCOMPUTING CHALLENGE
FINAL REPORT
APRIL 4, 2012

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1 Executive Summary

The aim of this project is to create an efficient parallel implementation of Ant Colony Optimization (**ACO**) applied to Traveling Salesman Problem (**TSP**). It should also be portable and easy to understand or modify. ACOs are algorithms based on ant foraging behavior. The TSP is a problem in which cities in an undirected graph must be connected by the shortest tour possible. A tour is a path that visits each city once and only once. ACOs have applications in problems including vehicle routing, networking, communications, and scheduling. **Data Parallelism** is a style of parallelism that usually consists of running the same fine grained operation for each piece of data in a very long vector.[6] The very large size of data that must be processed in a parallel ACO makes a data parallel implementation attractive. Due to the small size and large number of computations that must be performed at the same time, data parallelism particularly lends itself to computation on the Graphics Processing Unit (**GPU**). GPUs must do many tasks via "threads" in parallel to display pixels on the screen. They are convenient to use as general purpose processors for hardware acceleration of programs, as they are readily available on most computers. We used **Thrust**, a data-parallel C++ template library modeled off of the C++ Standard Library's[13] vector operations and implemented in a data parallel fashion, to implement our data-parallel ACO. Our implementation proved to be very effective. We achieved a speedup of about 100 (it varies with problem size) over a serial implementation. Our program has a computational complexity of $O(n \log(n))$ while the serial implementation has a complexity of $O(n^3)$. Excellants has made original contributions. Firstly, our implementation is portable to targets other than GPU. Also, we describe a tree algorithm that is important. Our code is also available and open source, both in this report and online. Additionally, our code is written in an easy to understand and modify C++ template library called Thrust. These advantages are important to anyone looking to use our work in a practical application or to extend it in the research world.

2 Problem Statement

Ants can forage for food quite efficiently. When an ant finds food, it leaves a pheromone trail back to the ant hill, which compels other ants to follow the same path. However, as the wind blows and the sun shines on this trail, the pheromones start to evaporate. Only the most traveled trails can continue to exist. Thus shorter, more popular paths are generated. Ant Colony Optimization (**ACO**) is a technique inspired by this ant foraging behavior, and can be used to generate good solutions to combinatorial optimization problems very quickly.[8]

Although Ant Colony Optimization seems more suited to foraging, it has proved itself a powerful metaheuristic that can be applied to problems ranging from routing to machine learning. However, ACOs are most conceptually suited to and commonly applied to the Traveling Salesman Problem (**TSP**). This is a very well-documented combinatorial optimization problem. In Symmetric TSP (referred to as TSP in this paper), n nodes in an undirected graph must be connected in the shortest tour possible. A **tour** is a path that visits each node once and only once. Each node is defined as a **city**, and a path connecting two cities is called a **route**. The TSP represents the dilemma of one unlucky salesman who has several cities to visit, but limited gas money and time in which he may do so. Our salesman would like to travel the shortest tour possible. Unfortunately, the TSP is a very difficult problem. A brute force approach to a TSP of n cities would have a computational complexity of

$$\frac{(n-1)!}{2} \tag{1}$$

Solving a 200-city TSP using brute force would take approximately 2.062×10^{360} years on Computer A [1], yet an ACO can get to within 2% of the optimum in 200 seconds. For the purposes of this project, the Travelling Salesman Problem will be used as a standard problem to solve, but is important to note that the TSP is not the focus of this project. Many excellent TSP solvers already exist. The focus is on ACO, which can be applied to many problems ranging from networking to protein folding, and TSP will be used as an example problem for ACO to tackle.

In some cases, the problem may change during the time an ACO is generating a solution. Let us use, as an example, the case of a truck driver delivering packages. He must deliver several hundred packages a day, and finding an optimal tour could take a while. To make matters worse, unan-

ticipated packages may arrive in the early morning. A fast ACO would save him time (less time spent waiting for his solution to be generated). It would also save money (if the optimization runs faster, using the time he has more efficiently, he would get a better tour and thus have to spend less money on gas). Thus, there are two advantages to having a faster ACO. Our driver's day is not over, however. Suppose there was an eleven-car pileup on a major road. The solution his program had generated was operating under the assumption that this road would be a convenient route, but now the driver needs a new tour. He could wait for the long simulation to run again, but if he had an ACO that could quickly generate a new solution based on previous calculations, he could get home to his family much faster.

This TSP with cities that change as the problem is being solved is called the Dynamic Travelling Salesman Problem (**DTSP**). The DTSP can be thought of in two ways. It could be that the problem changes after a solution has been generated, and the ACO simply resumes working with the previously calculated pheromones, or it could be thought that the problem changes as the ACO is solving it, and it must cope with the changes. The former situation being more suited toward a more stable real world application like our truck driver, and the latter being suited to something more volatile like a network routing problem. These two perspectives may arise out of different situations, but they are fundamentally identical in solution, in that the simulation must simply alter the data it has calculated before the change to fit the new conditions.

Clearly, an ACO applied to a DTSP would have to be fast, and could thus benefit from a parallel implementation. Implementing ACO in parallel is difficult, however, due to the random memory access patterns and the coordination of large parallel tasks.[8] Even though an ACO will not actually be applied to DTSP in this paper, it provides an excellent reason for an ACO to be sped up, and any advances in ACO techniques for a TSP could easily be applied to DTSP.

Due to the large amount of data that must be processed in an ACO and the relative simplicity of the computations that must be performed, a parallel implementation of ACO would be desirable. Also, because the TSP is simply a sample problem, the code of such an implementation would have to be simple enough to be modified for other problems. The aim of this project is to create an efficient parallel implementation of ACO on the GPU applied to TSP. It should also be portable and easily understood or modified.

3 Background

In order to understand the methods used to create an efficient parallel implementation of ACO, one must first understand the traditional implementation of an ACO applied to TSP. The mechanism of action for an ACO can be described as follows.

All ACOs have the same approximate structure. To initialize, they calculate all the distances between cities, make pheromone and probability matrices (a way to store the values of all the pheromones on all the trails), and create ants. Once the data is initialized, then the program enters the main loop in which the ants construct solutions and then lay pheromone based on the quality of these solutions.

In a TSP, ants start their tour construction at a random city. They then use probabilistic rules to decide where to move next until they have visited all the cities. Two factors influence these decisions. The first factor, τ_{ij} is the pheromone on a route from city i to city j . The second factor, η_{ij} is the inverse of the distance. The probability p_{ij}^k that ant k at city i will move to city j is given by:

$$p_{ij}^k = \frac{[\tau_{ij}]^\alpha [\eta_{ij}]^\beta}{\sum_{l \in N_i^k} [\tau_{il}]^\alpha [\eta_{il}]^\beta}, \quad i, j \in N_i^k \quad (2)$$

where N_i^k is a collection of all the cities the ant has not yet visited and α and β are parameters. Pheromone update is achieved in many different ways for different algorithms. In all cases, evaporation occurs on the routes first. The new amount of pheromone on a route τ'_{ij} is given by:

$$\tau'_{ij} = (1 - \rho)\tau_{ij} \quad (3)$$

where ρ is a parameter (from 0-1). Then, the ants deposit pheromone on their tours. Usually, the base unit of pheromone an ant lays down, $\Delta\tau_{ij}$, is given by:

$$\Delta\tau_{ij} = \frac{1}{C_{ij}} \quad (4)$$

where C_{ij} is the ant's tour length. After the ants deposit pheromone in some configuration, Pheromone update occurs in many ways, so the above equations are to help the reader understand the basic ways the pheromone update works.

3.1 Previous Work

For last year's Supercomputing Challenge, we created the most common implementations of ACO in Python. These implementations ran in parallel on the CPU using Python's multiprocessing module. We had great success with the performance enhancements that came from the parallelization of the algorithms. However, last year's program was not optimized for speed.[5] Designing last year's code inspired us to build a much faster, more optimized, more efficient version this year. All code from last year had to be scrapped as we were using a faster language and more powerful tools, including an entirely new approach to ACO parallelization.

4 Methods

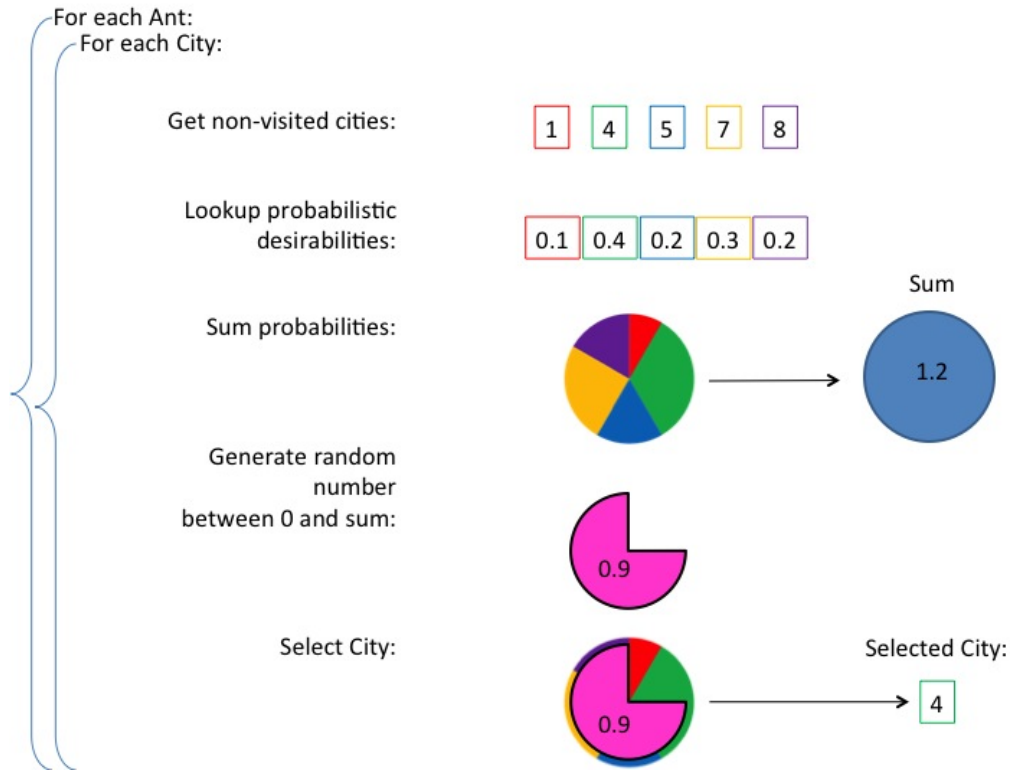
4.1 Tour Construction

Tour construction is the step in an ACO when all ants must construct paths that visit every city only once. Because tour construction takes up most of the time in an ACO[8], this is the aspect on which we focused most. Many implementations were tried and tested to find a suitable parallel tour construction method.

4.1.1 Serial Implementation

The probability of an ant at city i going into city j is described in Equation 2. In a traditional tsp, this probabilistic selection is accomplished through method analogous to a roulette wheel. The various probabilities that an ant may visit are gathered into a list. A random number is generated between 0 and the sum of these probabilities. The ant then iterates over each probability and selects its next city to visit. (One can imagine the roulette ball starting at the top and travels counter clockwise and ends up landing in a pie piece corresponding to a particular city) This process is repeated until an entire tour is created. See Figure 1. This will be very computationally costly as it is done for every ant for every city.

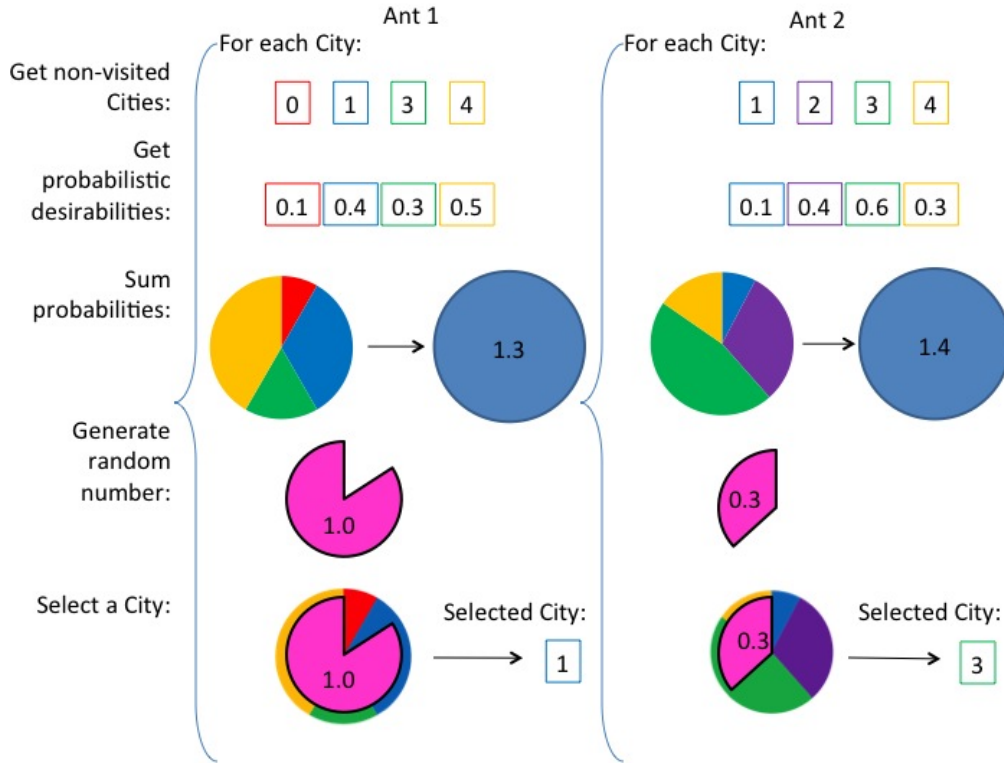
Figure 1: A typical serial implementation.



4.1.2 Task Parallel Implementation

Task Parallelism is the typical style of parallelism in which the parallelism is focused on doing different tasks on different pieces of data. Task parallelism usually consists of running processes to compute the results of more coarse grained tasks. In hopes of speeding up the traditional ACO, it would be tempting to simply run the ants in parallel. It does not scale efficiently, as each ant must still look at all of the next cities at every step of its tour sequentially ($O(n^2)$, where n is the number of cities). It also creates excessive overhead. All of the ants have random access patterns, the tasks may take different times to complete, and the size and number of tasks is also neither suited to a GPU nor CPU. Each processor must make an entire tour for one ant. The overhead significantly reduces the power of such an approach.[8] See Figure 2.

Figure 2: A typical task parallel implementation.



4.1.3 Turning to Data Parallelism

Data Parallelism is a style of parallelism focused on performing the same tasks on the same pieces of data. It usually consists of running the same fine-grained operation for each piece of data in a very long vector. [6] The very large size of data that must be processed in parallel makes a data parallel implementation attractive.[6] Due to the small size and large number of computations that must be performed at the same time, data parallelism particularly lends itself to computation on the Graphics Processing Unit (**GPU**). GPUs must do many tasks via "threads" in parallel to display pixels on the screen. They are convenient to use as general purpose processors for hardware acceleration of programs, as they are readily available on most computers. We chose to use Thrust [11], a data-parallel C++ template library modeled off of the C++ Standard Library's[13] vector operations and implemented in a data parallel fashion described by Guy Blelloch[6] in his

thesis, which has sometimes been described as a data-parallel bible. **Thrust** is very easy to use and modify for data-parallel operations. The most common Thrust target is CUDA, but it can also target OpenMP, OpenCL, or Thread Building Blocks. Thrust essentially translates the data parallel operations to primitive functions in CUDA, OpenMP, etc. It also provides host and device vector types, that store data on either the host or computation device. [11] Using Thrust eliminated the need for us to create a very specific and optimized data parallel functions and allowed us to create a more general ACO within the time allotted for this project. Some of these Thrust functions are used extensively in our code and assume a very important role in what we do. These functions are described by example in Table 1.

Table 1: Common primitive data-parallel functions supplied by Thrust.

Inputs		
A	0 5 1 8 7 3	
B	2 2 2 2 2 2	
C	1 2 5	
D	0 1 1 2 3 3 4 5	
Function	Output	Description
gather(C in A)	5 1 3	Index A by indicies in C
permutation_iterator(C in A)	5 1 3	Gather with kernel fusion
inclusive_scan(A)	0 5 6 14 21 24	Cumulative sum of A
transform(A and B with +)	2 7 3 10 9 5	Add A to B
reduce(A with +)	24	Sum A
sort(A)	0 1 3 5 7 8	Sort A
upper_bound(C in D)	3 4 7	Find last index of D where C could be inserted without violating ordering

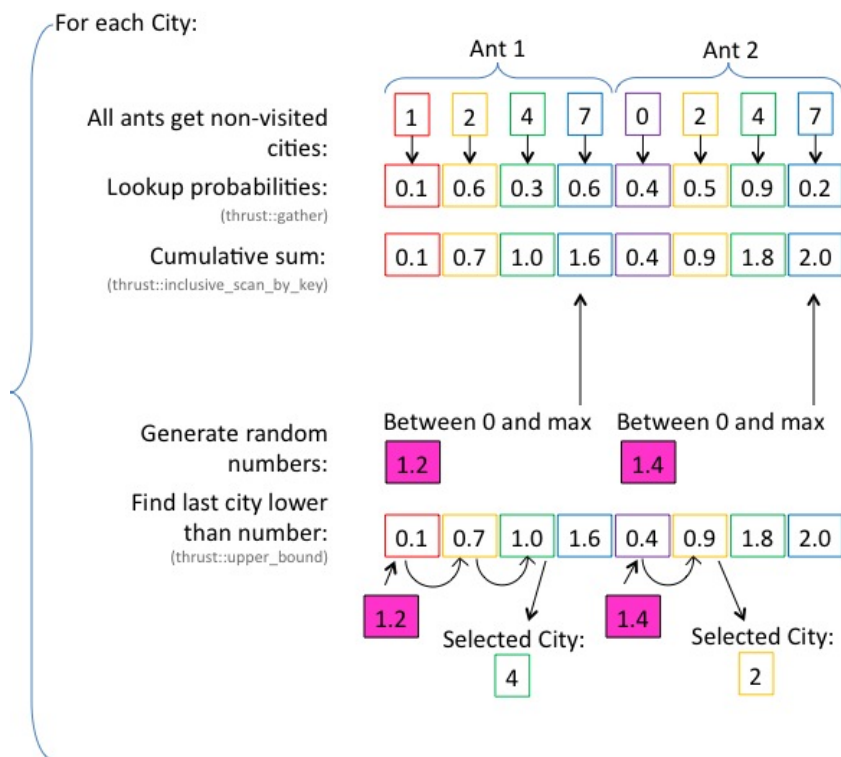
If a programmer can use Thrust functions as frequently and correctly as possible, their code will be both modular, portable, and efficient. For example, when performing multiple memory bound operations on a vector of data, it is better to use kernel fusion, or condense each of the operations to be performed into the same kernel, or chunk of code that will be executed on the device. Another example is the usage of Thrust functions such as the `zip_iterator` to create virtual arrays that can be processed without having to actually move or reorder large amounts of data.

4.1.4 Data Parallel Implementations

The first data parallel method we tested was analogous to stacking the previously described roulette wheels and selecting cities for every ant at the same time. Each ant gathered data for all the cities it was going to visit. This is very quick on a GPU because all of the lookups can be performed in parallel and there are many processors with which to do this. Then, a prefix sum(cumulative sum) was performed on a list of all the probabilities. This has the effect of evaluating each piece of the roulette wheel previously described in the traditional ACO at the same time. Then, the random numbers are generated, and the list of probabilities is iterated over in parallel by every ant.

This implementation suffers, however, due to the number of operations that must be performed in series. At every step of the tour construction, the cities the ant visited had to be updated, the probability gathered, the probabilities prefix summed, random number bounds selected, random numbers generated, and searches performed. While this implementation was fairly straightforward to code and understand it needed to be improved upon. See Figure 3.

Figure 3: Our initial data parallel implementation.



A new method had to be implemented that was completely different from all the others. More of the operations had to be grouped and performed at the same time. To accomplish this a tree-based algorithm was implemented. All the probabilities are gathered as previously described for each ant. Then, each probability is assigned to a thread, along with the city it is associated with, and a random number. At each step in the tree, two cities are reduced to one. The random number is used to probabilistically select a city based on the probabilities given. Then, the probabilities are summed and the chosen city is given to the next level of the tree. The unused random number is given to the next level of the tree. This mathematically can select from a large list of cities a single city randomly with a bias toward the probabilities in the same way that the previous algorithms have described. The reason the probabilities are summed has to do with multiplication of probabilities. The probability of going from city i to city j should be equivalent to the desirability metric of that city divided by the sum of the probabilities of all

the other cities that ant can visit. The probability of an example city 1 being selected out of five cities in the tree algorithm is shown below in Equation 5. The probability of city 1 being selected is equivalent to the probability of city 1 being selected at every level of the tree.

$$p_{5\ 1}^k = \frac{p_1}{p_1 + p_2} * \frac{p_1 + p_2}{p_1 + p_2 + p_3 + p_4} = \frac{[\tau_{ij}]^\alpha [\eta_{ij}]^\beta}{\sum_{l \in N_i^k} [\tau_{il}]^\alpha [\eta_{il}]^\beta} \quad (5)$$

As one can see, the probabilities simplify to Equation 2 as desired. This implementation performed very well and can also scale efficiently. The reduction can perform city selection for all of the ants and all of the cities in parallel for every level of the tree, making the reduction $O(\log(n))$. The reduction must be performed for every city, so the whole tour construction step is $O(n \log(n))$. The implementation is described in the diagram below, and pseudocode is given for the decision function at each node in the tree. See Algorithm 1, Figure 4.

Algorithm 1 The algorithm used to reduce two cities in the tree selection method.

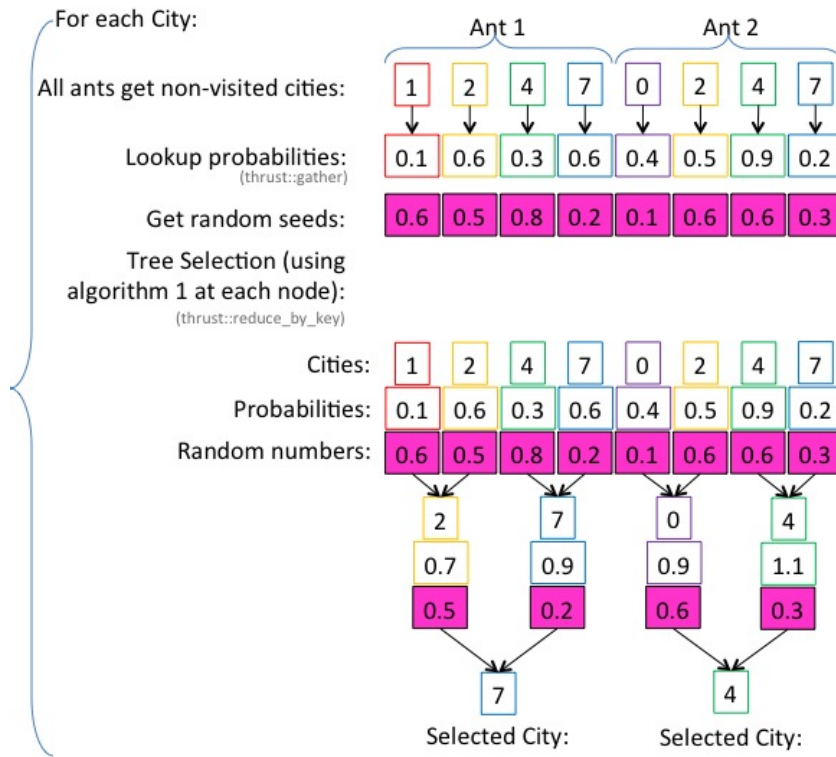
Inputs: (City A, Probability A, Random A),
(City B, Probability B, Random B)

```

if (Random A * (Probability A + Probability B) < Probability A)
{
    return (City A,
            (Probability A + Probability B), // Probabilities added
            Random B) // Unused random returned
} else {
    return (City B,
            (Probability A + Probability B), // Probabilities added
            Random B) // Unused random returned
}

```

Figure 4: Our best data parallel implementation.



4.2 Pheromone Update

Even though the tour construction is the most time consuming, it was simple enough to implement pheromone updates in parallel, and important to do as it must not become a bottleneck. It is also advantageous to do this on a GPU as data transfer between the GPU and the CPU is expensive in terms of computation time.

The pheromone update is straightforward to implement but can be different for every ACO variant. As a general rule, it makes sense to add the previously calculated amount of pheromone to each route in the necessary tours. For our project we settled on a Rank Based Ant System as it is simple to implement. In a Rank Based Ant System, ants deposit pheromone according to their rank. Their rank is assigned based on the quality of their tour. Most of the time the number of ranks is usually around six so only six ants will deposit pheromone. Thankfully, Thrust has a very well implemented

and efficient sort, so this sort was used to assign the six ranks.

4.3 Probability Calculation

Calculating probabilities is also simple to do in parallel. Each probability for a particular route is calculated by a separate thread, using the distance of that route and the pheromone on that route. Probability calculations for each route are easily parallelized. Each probability is calculated in parallel. The distance and pheromone for that particular route can be looked up by a separate thread, making this a very simple and quick implementation. `thrust::transform()` was used to calculate Equation 2 ($probability = pheromone^\alpha * 1/distance^\beta$) for all of the cities in parallel. See Figure 5

Figure 5: Probability calculation with `thrust::transform()`.

Pheromones		Distances		Probabilities
0.2^α	X	4^β	=	<input type="text"/>
0.6^α	X	3^β	=	<input type="text"/>
0.1^α	X	5^β	=	<input type="text"/>

5 Results

It was initially decided that Marco Dorigo’s code would be used as a benchmark test against which to compare our code.[8] This, however, led to issues because his code did not allow for large numbers of ants (i.e. greater than 100). For this reason a new ACO was selected. We settled on libaco[10] because it is a very standard ACO, and can accommodate the large number of ants that we require. It is as accurate as Dorigo’s, but may be slower. We cannot measure because Dorigo’s code does not allow for large numbers of ants. The exact speed may not be as important of a metric as how the algorithm scales. This is because the two algorithms are being run on different devices. For this reason, both experiments measuring speedup and experiments validating theoretical scaling have been run. There is, however, a limit to scaling, as the number of threads and amount of local memory on a GPU may reach an upper limit. Another metric used to assess the

quality of both algorithms is a comparison of the quality of the tour after a certain amount of time. This is a similar metric to a simple speedup calculation, but is looking at a fixed time limit instead of a fixed quality (same number of mathematically identical iterations) as shown above. These two metrics address both sides of the time/quality trade-off described in detail in the problem statement. All statistical calculations were done with R, a statistical language.[9]

5.1 Validation

5.1.1 Experiment 1

The first challenge was to validate our ACO. Even though the two implementations were created to produce equal results, we felt that evidence of this should be given. To do this, we compare the tour lengths after 20 iterations from 20 trials of both implementations on Computer A[1] on dj38.tsp from National TSP [3]. This is a simple test problem of the largest population centers in Djibouti. A two sample t-test of these 20 trials yields a p-value of .638. Thus, assuming the programs produce identical results, the probability that the discrepancies in the results we obtained were due to chance is 63.8%. This is likely enough to assume that the two implementations produce identical results. Knowing that the implementation is valid, tests can be run to determine scaling capability.

5.2 Scaling and Speedup

An ACO could scale with respect to a few metrics. For clarity, the number of cities will be referred to by the parameter n . The number of ants will be referred to by the parameter m .

5.2.1 Experiment 2

The first test of scaling capability was done with respect to the number of cities. The number of ants was held at a constant 128. The city sets for this test were the first n cities of usa13509.tsp[12], a set of cities in the United States. This and all following scaling tests were performed on Computer B[2]. The results are shown for both implementations in Table 3. Note that a maximum speedup of 100 was achieved.

Table 2: The time to complete one iteration with respect to the number of cities.

Number of Cities	libaco Time (s)	ExcellAnts Time (s)	Speedup
32	0.0675	0.0138	4.89
64	0.2881	0.0254	11.34
128	1.2141	0.0630	19.27
256	5.2334	0.1344	39.35
512	22.3871	0.3707	60.39
1024	95.1977	0.9484	100.38

The expected scaling of a serial implementation with respect to cities should be $O(n^2)$, as each city must be examined at every city in the tour. With this assumption, a fitted linear regression was performed on the data. The correlation constant was 1.000, and the residuals were scattered. r^2 was 0.9998, meaning that 99.98% of the variation in the data is explained by the theoretical scaling. This means that we can safely say the above model is correct.

The expected scaling of our data-parallel implementation with respect to cities should be $O(n \log(n))$, as all the cities are reduced (in $\log(n)$ time) at every city in the tour. With this assumption, a fitted linear regression was performed on the data. The correlation constant was 0.9973, and the residuals were scattered. r^2 was 0.9947, meaning that 99.47% of the variation in the data is explained by the theoretical scaling. This means that we can safely say the above model is correct.

5.2.2 Experiment 3

The second test of scaling capability used different numbers of ants with the same number of cities. The city set for this problem was held at the constant first 128 cities of `usa13509.tsp`[12]. The results are shown for both implementations in Table 3. Note that we achieved a maximum speedup of 81.

Table 3: The time to complete one iteration with respect to the number of ants.

Number of Ants	libaco Time (s)	ExcellAnts Time (s)	Speedup
32	0.3032	0.0484	6.26
64	0.6077	0.0532	11.42
128	1.2124	0.0628	19.31
256	2.4238	0.0642	37.75
512	4.842	0.0912	53.09
1024	9.6713	0.1184	81.68

The expected scaling of a serial implementation with respect to ants should be $O(m)$, as the tour construction is performed for every ant. With this assumption, a fitted linear regression was performed on the data. The correlation constant was 1.000. The residuals were patterned, but they were too small to acknowledge. r^2 was 0.9999, meaning that 99.99% of the variation in the data is explained by the theoretical scaling. This means that we can safely say the above model is correct.

The expected scaling of data-parallel implementation with respect to ants should be $O(\log(m))$, as all the ants' possible cities are reduced (in $\log(n)$ time) for a constant amount of cities. With this assumption, a fitted linear regression was performed on the data. The correlation constant was 0.9310. r^2 was 0.8665, meaning that 86.65% of the variation in the data is explained by the theoretical scaling. Although this is a relatively low r^2 value, the residuals are very scattered, and show absolutely no clear pattern. This means that we can safely say the above model is correct.

5.2.3 Experiment 4

The third test of scaling capability considered both ants and cities. The number of ants was set to the recommended amount, the number of cities[8]. The city sets for this test were the first n cities of usa13509.tsp[12]. The results are shown for both implementations graphically in figures 6 and 7. Note that the vertical axes have different values. Note that a maximum speedup of 340 was achieved.

Table 4: The time to complete one iteration with respect to the number of cities and ants.

Number of Cities and Ants (n = m)	libaco Time (s)	ExcellAnts Time (s)	Speedup
32	0.0169	0.0099	1.71
64	0.1429	0.0281	5.09
128	1.2123	0.0631	19.21
256	10.4507	0.1861	56.16
512	89.6375	0.6784	132.13
1024	756.8478	2.2204	340.86

Figure 6: The time for libaco to complete one iteration with respect to the number of cities and ants.

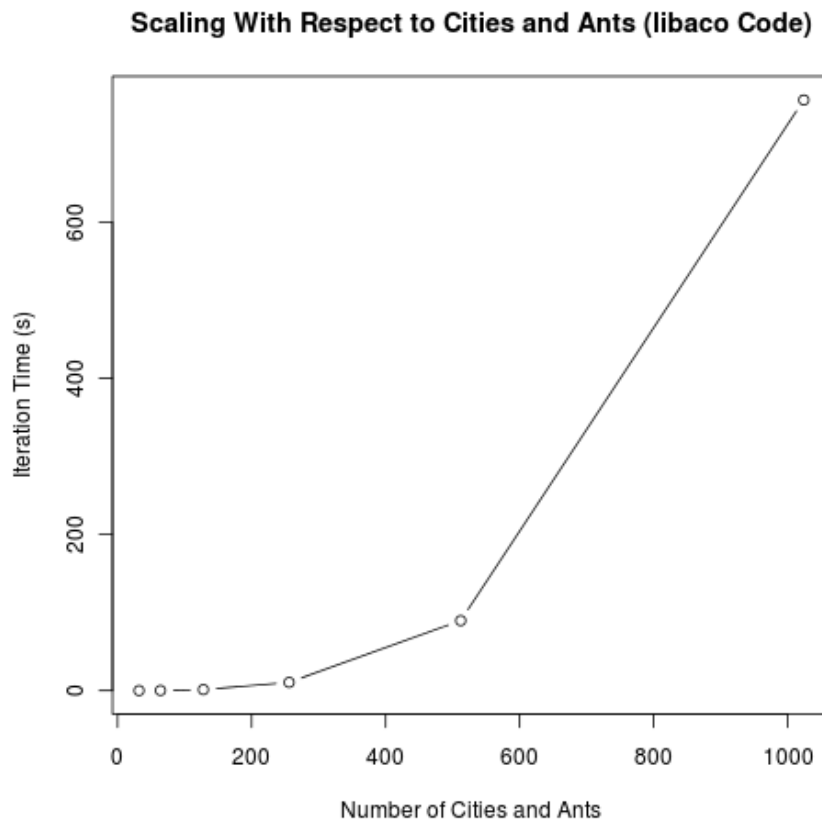
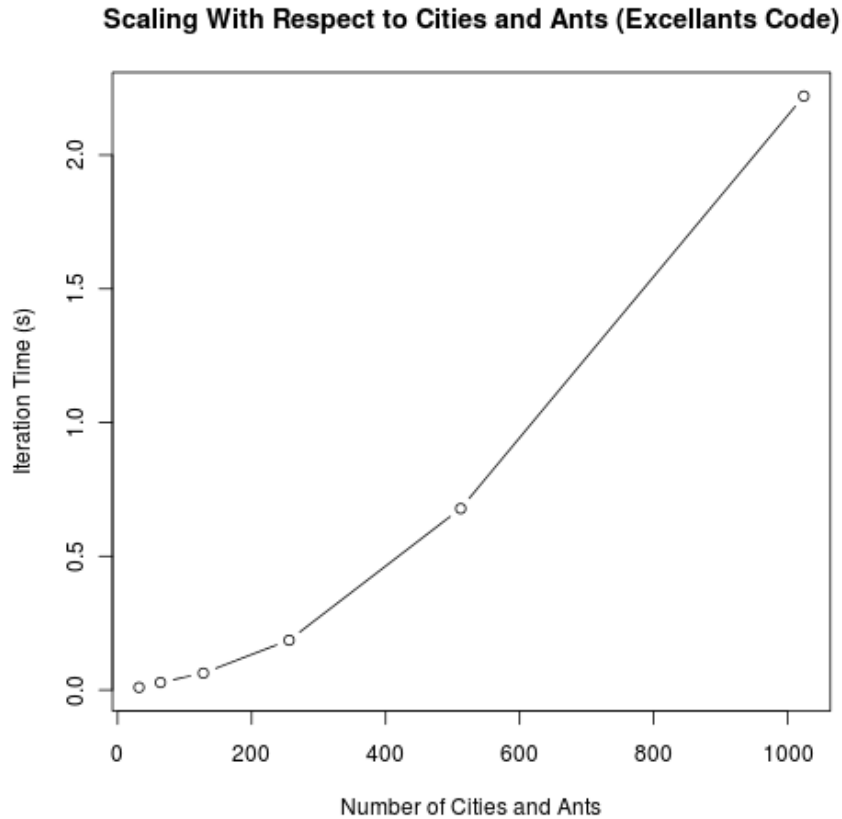


Figure 7: The time for ExcellAnts to complete one iteration with respect to the number of cities and ants.



The expected scaling of a serial implementation with respect to cities and ants should be $O(n^2m)$ or $O(n^3)$ ($n = m$), as each city must be examined at every city in each ant's tour. With this assumption, a fitted linear regression was performed on the data. The correlation constant was 1.000, and the residuals were scattered. r^2 was 0.9999, meaning that 99.99% of the variation in the data is explained by the theoretical scaling. This means that we can safely say the above model is correct. The expected scaling of our data-parallel implementation with respect to cities and ants should be $O(n \log(n))$ as all the cities for all the ants are reduced (in $\log(n)$ time) for every city in the tour. This is the same scaling equation that we saw in Experiment 2 for the serial code. The reason for this is that although the cities must be

evaluated for every ant, they are all reduced at the same time, producing $O(n \log(nm))$, or $O(n \log(n^2))$ ($n = m$), which reduces to $O(n \log(n))$. With this assumption, a fitted linear regression was performed on the data. The correlation constant was 0.9982, and the residuals were scattered. r^2 was 0.9765, meaning that 97.65% of the variation in the data is explained by the theoretical scaling. This means that we can safely say the above model is correct.

5.3 Quality

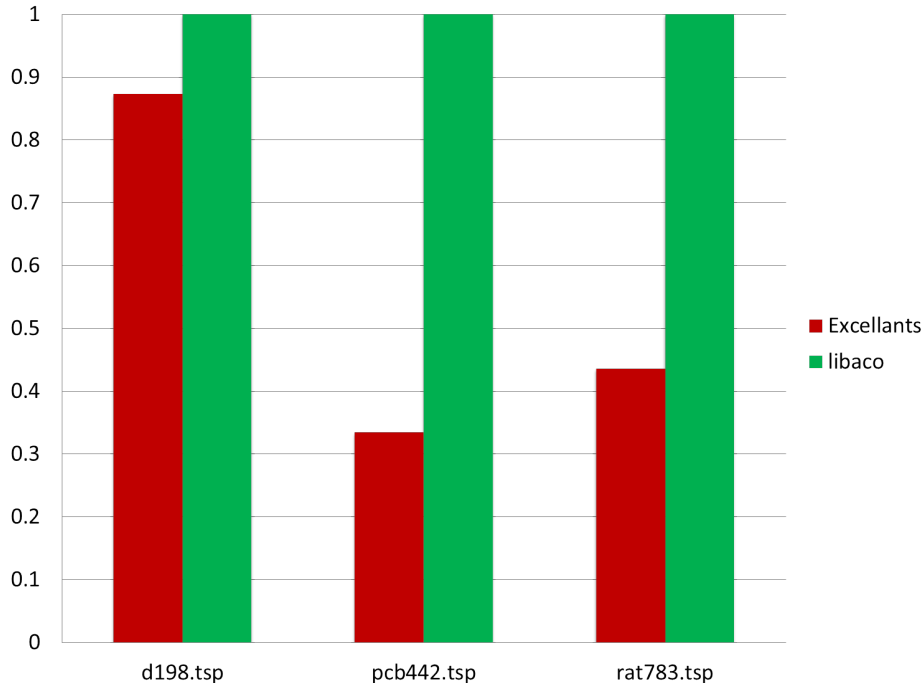
5.3.1 Experiment 5

One could think of performance as getting a faster solution or getting a better solution. In this experiment, the two implementations were compared by their end tour quality. The following tests were run on Computer A.[1] Both libaco and ExcellAnts tour qualities were measured at the end of 100 seconds on different datasets from TSPLIB. [12] The results are presented in Table 5 and the proportional improvement can be visually compared in Figure 8. The Excellants quality was up to three times better.

Table 5: The best tour lengths generated by each ACO in 100 seconds.

TSP	d198.tsp	pcb442.tsp	rat783.tsp
ExcellAnts	16536	64727.6	3165.93
libaco	18941	193819	7263.16
libaco/ExcellAnts	0.87	0.33	0.44

Figure 8: The proportional difference in best tour lengths generated by each ACO in 100 seconds.



6 Conclusions

A data-parallel ACO on the GPU that is easy to program and is portable to multiple targets has its advantages. First and foremost, it's faster. The parallel code outperformed the serial code in all cases.(see Tables 3, 2, 4) We achieved a speedup over the libaco code of about 100 with 512 ants and 512 cities. The speed can also lead to better quality. In some cases, ExcellAnts achieved three times the quality of a serial implementation. Also, it scales much more efficiently. If one ran their simulations at the recommended number of ants, libaco would scale at $O(n^3)$ while ExcellAnts would scale at $O(n \log(n))$. This project successfully created a working data-parallel implementation of ACO that significantly outperformed the serial version, with vast improvements in scaling capability. Even a simple task-parallel approach would only reach a scaling of $O(n^2)$ with respect to cities and ants, as all cities must still be evaluated at each city in the tour. Our program was

also portable to different targets. We were able to run our program with OpenMP. Our code also made use of an off the shelf data parallel library, for ease of programming.

7 Significant Original Achievement

ExcellAnts has made some important contributions. Firstly, we wrote an efficient, high-quality data-parallel implementation of ACO (via Thrust [11]) that is portable to multiple targets (CUDA, OpenCL, OpenMP, or Thread Building Blocks) without having to change the code. Secondly, the methods used to construct the tree-based selection process are described in detail. This is important to anyone attempting their own implementation using our methods. Thirdly, our code is also available and open source, both in this report and online.[4] Fourth, the time to build this efficient, portable implementation was less than what it would have taken to create a hand-tuned implementation on each target, since Thrust provides a library that is easy to use. These advantages are important to anyone looking to use our work in a practical application or to extend it in the research world.

8 Related Work

It came to our attention as this paper was being written that a paper on a hand-tuned GPU implementation of ACO was available online in January 2012.[7] However, all the original ideas presented here were conceived independently from this work. We did not read it until we had done our implementation and started writing our paper. We have had several original achievements, and actually some improvements over this work. The first one of these is the tree-based selection process. This sped up our code considerably and also allows it to scale to larger data sets more efficiently. Ours is described in detail, their's is not. The other methods initially attempted were also of our own design. Our code is also written in Thrust, while theirs is written in CUDA and highly optimized to a GPU, making it difficult to understand and non-portable. While their code is not currently available, The code for ExcellAnts is available in an open source format online.[4]

9 Future Work

This report does not mark the end of our project. We aim to continue our work on our Google Project page.[4] We have several plans which we expect to apply to the code in the future. One of the most prominent of these is that we plan to use our ACO on a dynamic traveling salesman problem (DTSP) and optimize it for use with DSTPs. Another plan is to overlap CPU and GPU computation by running suitable operations on both processors in order to maximize the use of the computational power of the system on which it is running. Yet another goal for us to work toward in the future is to add more implementations of ACO to give the user more flexibility.

10 Work Products

10.1 Code

```
1  /*****
2  * Setup.cpp
3  * Peter Ahrens
4  * Sets up the ACO
5  *****/
6
7  #include "RankBasedAntSystem.h"
8  #include "TSPReader.h"
9  #include "Comm.h"
10 #include "Writer.h"
11 #include <iostream>
12 #include <unistd.h>
13 #include <string>
14 #include <cctype>
15 #include <ctime>
16 using namespace std;
17
18 //Setup: The main control loop to the whole program.
19 int main(int argc, char* argv[]) {
20     //declare variables
21     cout << "|SETUP|\n";
22     string antHillType = "RBAS";
23     int m = -1;
24     int ranks = 6;
25     int maxTime = 0;
```



```

26 int maxIter = 0;
27 int maxReps = 0;
28 int reps = 0;
29 bool stopping = false;
30 bool graphics = false;
31 char* filen;
32 Writer O; //writes output to stdout and an optional file
33 //Read initially necessary command-line arguments.
34 for(int i = 0; i < argc;i++){
35     if (string(argv[i]) == "-ras"){
36         antHillType = "RBAS";
37         if(i + 1 < argc){
38             if(string(argv[i])[0] != '-'){
39                 ranks = atoi(argv[i+1]);
40             }
41         }
42     }
43     if (string(argv[i]) == "-tsp"){
44         filen = argv[i+1];
45     }
46     if (string(argv[i]) == "-gui"){
47         graphics = true;
48     }
49     if (string(argv[i]) == "-m"){
50         m = atoi(argv[i+1]);
51     }
52     if (string(argv[i]) == "-out"){
53         if(!O.setFile(argv[i+1])){
54             cout << "Unable to open output file\n";
55         }
56     }
57     if (string(argv[i]) == "-maxTime"){
58         maxTime = atoi(argv[i+1]);
59     }
60     if (string(argv[i]) == "-maxIter"){
61         maxIter = atoi(argv[i+1]);
62     }
63     if (string(argv[i]) == "-maxReps"){
64         maxReps = atoi(argv[i+1]);
65     }
66 }
67 cout << ">" << flush; //——Checkpoint 1
68 if(graphics){
69     //If graphics are running, create pipes and processes.
70     int parentPipe [] = {-1,-1}; // parent -> child

```

```

71     int childPipe [] = {-1,-1}; // child -> parent
72     if ( pipe(parentPipe) < 0 || pipe(childPipe) < 0 )
73     {
74         std::cout << "Failed to create pipe";
75         return 1;
76     }
77     #define PARENT_READ    childPipe[0]
78     #define CHILD_WRITE    childPipe[1]
79     #define CHILD_READ    parentPipe[0]
80     #define PARENT_WRITE  parentPipe[1]
81     pid_t pID = fork();
82     if(pID == 0){//child
83         close(PARENT_WRITE);
84         close(PARENT_READ);
85         //Graphics run in a separate process from the computations
86         //so neither is slowed down.
87         Comm C(CHILD_READ,CHILD_WRITE);
88         cout << ">" << flush; //——Checkpoint 3
89         if(!C.send(string("Test"))){
90             cout << "\n Interprocess Comm Failed";
91             return 1;
92         }
93     }else if(pID < 0){//fail
94         cout << "Failed to fork";
95         return 1;
96     }else{//parent
97         close(CHILD_READ);
98         close(CHILD_WRITE);
99         //Computations run in a separate process from the graphics
100        //so neither is slowed down.
101        Comm C(PARENT_READ,PARENT_WRITE);
102        cout << ">" << flush; //——Checkpoint 2
103        TSPReader t;
104        t.read( filen );
105        cout << ">" << flush; //——Checkpoint 4
106        if(m == -1){
107            m = t.getNumNodes();
108        }
109        RankBasedAntSystem antHill(t.getDistances(),t.getNumNodes
110        (),m);
111        cout << ">" << flush; //——Checkpoint 5
112        //If any parameters need to be changed, they are modified
113        //from their defaults here.
114        for(int i = 0; i < argc;i++){
115            if (string(argv[i]) == "-b"){

```

```

112     antHill.setBeta(atof(argv[i+1]));
113 }
114 if (string(argv[i]) == "-r"){
115     antHill.setRho(atof(argv[i+1]));
116 }
117     }
118     cout << ">" << flush; //——Checkpoint 6
119     antHill.initialize();
120     if(C.recieve() == "Test"){ //A check to find out if the
121         comm is working.
122     }else{
123     cout << ">\n" << flush; //——Checkpoint 7
124     return 1;
125     }
126     O.writeHeader(antHill.getBeta(), antHill.getRho(), antHill.
127         getNumAnts(), antHillType, t.getName());
128     clock_t t1, t2, t3;
129     t1 = t2 = t3 = clock();
130     //Main control sequence.
131     for(int i = 0; !stopping; i++){
132     t2 = t3;
133     antHill.forage();
134     t3 = clock();
135     O.write(i, antHill.getIterBestDist(), antHill.getGlobBestDist
136         (), (double)(t3 - t1) / CLOCKS_PER_SEC, (double)(t3 - t2) /
137         CLOCKS_PER_SEC);
138     if(maxTime != 0){
139     if(((int)((double)(t3 - t1) / CLOCKS_PER_SEC)> maxTime){
140     stopping = true;
141     }
142     }
143     if(maxIter != 0){
144     if(i >= maxIter){
145     stopping = true;
146     }
147     }
148     if(maxReps != 0){
149     if(antHill.getReps() >= maxReps){
150     stopping = true;
151     }
152     }
153     }else{

```

```

153 cout << ">" << flush; //——Checkpoint 2
154 TSPReader t;
155 t.read( file );
156 cout << ">" << flush; //——Checkpoint 3
157 if( m == -1 ){
158     m = t.getNumNodes();
159 }
160 RankBasedAntSystem antHill( t.getDistances(), t.getNumNodes(),
161     m );
162 //If any parameters need to be changed, they are modified
163 //from their defaults here.
164 cout << ">" << flush; //——Checkpoint 4
165 for( int i = 0; i < argc; i++ ){
166     if( string( argv[ i ] ) == "-b" ){
167         antHill.setBeta( atof( argv[ i+1 ] ) );
168     }
169     if( string( argv[ i ] ) == "-r" ){
170         antHill.setRho( atof( argv[ i+1 ] ) );
171     }
172 }
173 cout << ">" << flush; //——Checkpoint 5
174 antHill.initialize();
175 cout << ">>\n" << flush; //——Checkpoint 6/7
176 O.writeHeader( antHill.getBeta(), antHill.getRho(), antHill.
177     getNumAnts(), antHillType, t.getName() );
178 clock_t t1, t2, t3;
179 t1 = t2 = t3 = clock();
180 //Main control sequence.
181 for( int i = 0; !stopping; i++ ){
182     t2 = t3;
183     antHill.forage();
184     t3 = clock();
185     O.write( i, antHill.getIterBestDist(), antHill.
186         getGlobBestDist(), (double)(t3 - t1) / CLOCKS_PER_SEC,
187         (double)(t3 - t2) / CLOCKS_PER_SEC );
188     if( maxTime != 0 ){
189         if( (double)(t3 - t1) / CLOCKS_PER_SEC > maxTime ){
190             stopping = true;
191         }
192     }
193     if( maxIter != 0 ){
194         if( i >= maxIter ){
195             stopping = true;
196         }
197     }
198 }

```

```

193     if(maxReps != 0){
194     if(antHill.getReps() >= maxReps){
195         stopping = true;
196     }
197     }
198 }
199 }
200 }
201
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212
213
214 /*****
215 * RankBasedAntSystem.h
    *****/

```

```

216 * Peter Ahrens *
217 * Performs specific RBAS procedures *
218 *****/
219
220 #ifndef RANKBASEDANTSYSTEMH
221 #define RANKBASEDANTSYSTEMH
222 #include "Colony.h"
223
224 //RankBasedAntSystem: Provides the necessary extensions to
    Colony to create a Rank-Based Ant System
225 class RankBasedAntSystem : Colony{
226 public:
227     RankBasedAntSystem(thrust::host_vector<float> newDistances ,
        int newNumCities, int newNumAnts); // Allocates memory and
        sets defaults.
228     void initialize(); // Runs the Colony initialize, then creates
        additional maps and keys.
229     void computeParameters(); // Simply computes necessary
        parameters.
230     void forage(); // Runs Colony forage.
231     void setRho(float newRho);
232     void setBeta(float newBeta);
233     void setW(int newW);
234     int getW();
235     double getRho();
236     double getBeta();
237     int getNumAnts();
238     double getIterBestDist();
239     double getGlobBestDist();
240     int getReps();
241 private:
242     void computeInitialPheromone(); // Computes the initial
        pheromone level with the formula described by Marco Dorigo.
243     void updatePheromones(); // Evaporates, then the ants lay
        pheromone at levels corresponding to their rank, judged by
        the distances of their tours.
244     int w;
245     thrust::device_vector<float> RBASWeight;
246     thrust::device_vector<int> RBASMap;
247 };
248
249 #endif
250
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```

```

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261
262
263 /*****
264  * RankBasedAntSystem.cu          *
265  * Peter Ahrens                  *
266  * Performs specific RBAS procedures *
267  *****/
268
269 #include "RankBasedAntSystem.h"
270
271 //constructor: Allocates memory and sets defaults.
272 RankBasedAntSystem::RankBasedAntSystem(thrust::host_vector<float
    > newDistances, int newNumCities, int newNumAnts)
273 : Colony(newDistances, newNumCities, newNumAnts)
274 {

```

```

275     w = 6; // default
276     RBASWeight = thrust::device_vector<float>(numAnts);
277     RBASMap = thrust::device_vector<int>(numAnts*numCities);
278 }
279
280 //initialize: Runs the Colony initialize , then creates
           additional maps and keys.
281 void RankBasedAntSystem::initialize ()
282 {
283     Colony::initialize ();
284     thrust::fill (RBASWeight.begin () ,RBASWeight.end () ,0);
285     thrust::copy_n(thrust::make_reverse_iterator (thrust::
           make_counting_iterator (w)),
286                   w,
287                   RBASWeight.begin ());
288     thrust::fill (ACInt.begin () ,
289                  ACInt.end () ,
290                  0);
291     ACInt[numCities] = 1;
292     thrust::inclusive_scan (ACInt.begin () ,
293                             ACInt.end () ,
294                             ACInt.begin ());
295     thrust::exclusive_scan_by_key (ACInt.begin () ,
296                                    ACInt.end () ,
297                                    thrust::make_constant_iterator (1,0) ,
298                                    RBASMap.begin ());
299 }
300
301 //computeParameters: Simply computes necessary parameters.
302 void RankBasedAntSystem::computeParameters ()
303 {
304     if (numCities < w){
305         w = numCities;
306     }
307     computeInitialPheromone ();
308 }
309
310 //computeInitialPheromone: Computes the initial pheromone level
           with the formula described by Marco Dorigo.
311 void RankBasedAntSystem::computeInitialPheromone ()
312 {
313     initialPheromone = 0.5*w*(w-1)/(rho * Colony::greedyDistance ()
           );
314 }
315

```



```

316 //updatePheromones: Evaporates, then the ants lay pheromone at
    levels corresponding to their rank, judged by the distances
    of their tours.
317 void RankBasedAntSystem::updatePheromones()
318 {
319     //evaporate
320     thrust::transform(pheromones.begin(),
321                     pheromones.end(),
322                     thrust::make_constant_iterator(1.0f-rho),
323                     pheromones.begin(),
324                     thrust::multiplies<float>());
325     //determine ant pheromone levels
326     thrust::stable_sort_by_key(thrust::make_permutation_iterator(
        antDistances.begin(),ACKey.begin()),
327                               thrust::make_permutation_iterator(antDistances.end(),
        ACKey.end()),
328                               antTours.begin());
329     thrust::transform(RBASWeight.begin(),
330                     RBASWeight.end(),
331                     antDistances.begin(),
332                     AFloat.begin(),
333                     thrust::divides<float>());
334     //AFloat[w-1] = w/globBestDist;
335     AFloat[w-1] = w/iterBestDist;//for a simple rankbased, without
        global pheromone
336     ACInt.assign(antTours.begin(),antTours.end());
337     //thrust::scatter(globBestTour.begin(),globBestTour.end(),
        thrust::make_counting_iterator(numCities*(w-1)),ACInt.begin
        ());
338     thrust::scatter(iterBestTour.begin(),
339                     iterBestTour.end(),
340                     thrust::make_counting_iterator(numCities*(w-1)),
341                     ACInt.begin()); //for a simple rankbased, without global
        pheromone
342     thrust::transform(ACInt.begin(),
343                     ACInt.end(),
344                     thrust::make_permutation_iterator(ACInt.begin(),distMap.
        begin()),
345                     ACInt2.begin(),
346                     saxpy_functor(numCities));
347     //lay Pheromone
348     for(int i = 0; i < numCities*w; i += numCities){
349         thrust::transform(thrust::make_permutation_iterator(
            pheromones.begin(),ACInt2.begin() + i),

```

```

350         thrust::make_permutation_iterator(pheromones.end(),
351             ACInt2.begin() + i + numCities),
352         thrust::make_permutation_iterator(AFloat.begin(), ACKey
353             .begin() + i),
354         thrust::make_permutation_iterator(pheromones.begin(),
355             ACInt2.begin() + i), thrust::plus<float>());
356     }
357 }
358
359 //forage: Runs Colony forage.
360 void RankBasedAntSystem::forage()
361 {
362     Colony::forage();
363 }
364
365 void RankBasedAntSystem::setRho(float newRho)
366 {
367     Colony::setRho(newRho);
368 }
369
370 void RankBasedAntSystem::setBeta(float newBeta)
371 {
372     Colony::setBeta(newBeta);
373 }
374
375 void RankBasedAntSystem::setW(int newW)
376 {
377     w = newW;
378 }
379
380 int RankBasedAntSystem::getW()
381 {
382     return w;
383 }
384
385 double RankBasedAntSystem::getRho()
386 {
387     return Colony::getRho();
388 }
389
390 double RankBasedAntSystem::getBeta()
391 {
392     return Colony::getBeta();
393 }

```

```

392 int RankBasedAntSystem::getNumAnts()
393 {
394     return Colony::getNumAnts();
395 }
396
397 double RankBasedAntSystem::getIterBestDist()
398 {
399     return Colony::getIterBestDist();
400 }
401
402 double RankBasedAntSystem::getGlobBestDist()
403 {
404     return Colony::getGlobBestDist();
405 }
406
407 int RankBasedAntSystem::getReps()
408 {
409     return Colony::getReps();
410 }
411
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422
423
424 /*****
425  * Colony.h                               *
426  * Peter Ahrens                           *
427  * Main ACO procedures                     *
428  *****/
429
430 #ifndef COLONY_H
431 #define COLONY_H
432 #include <thrust/host_vector.h>
433 #include <thrust/device_vector.h>
434 #include <thrust/copy.h>
435 #include <thrust/fill.h>
436 #include <thrust/binary_search.h>
437 #include <thrust/sequence.h>
438 #include <thrust/adjacent_difference.h>
439 #include <thrust/random.h>
440 #include <thrust/functional.h>
441 #include <thrust/gather.h>
442 #include <thrust/iterator/constant_iterator.h>
443 #include <thrust/iterator/discard_iterator.h>
444 #include <thrust/scan.h>
445 #include <thrust/sort.h>
446 #include <thrust/remove.h>
447 #include <iostream>
448 #include <sys/time.h>
449 #include <math.h>
450 //saxpy_functor: Performs the operation  $s = a * x + y$ , where a
    is a constant.
451 struct saxpy_functor
452 {
453     const float a ;
454     saxpy_functor ( float _a ) : a ( _a ) {}
455     __host__ __device__
456     float operator () ( const float & x , const float & y ) const
457     {
458         return a * x + y ;

```

```

459 }
460 };
461
462 //treeSelect: The main function used to reduce two cities that
463 //an ant might visit.
464 struct treeSelect : public thrust::binary_function<thrust::tuple
465 <int ,float , unsigned int >,thrust::tuple<int ,float , unsigned int
466 >,thrust::tuple<int ,float , unsigned int > >
467 {
468     treeSelect() {}
469     __host__ __device__
470     thrust::tuple<int ,float , unsigned int> operator()(const
471     thrust::tuple<int ,float , unsigned int> tup1,const thrust::
472     tuple<int ,float , unsigned int> tup2) const
473     {
474         const float prob = tup1.get<1>() + tup2.get<1>();
475         if(tup1.get<2>() * prob / 4294967296 < tup1.get<1>()){
476             return thrust::make_tuple(tup1.get<0>(),prob,tup2.get<2>()
477             );
478         } else{
479             return thrust::make_tuple(tup2.get<0>(),prob,tup2.get<2>()
480             );
481         }
482     }
483 };
484
485 //prob_functor: Performs the probabilistic desirability
486 //calculation  $s = (\text{pheromone level})^\alpha * (1/\text{distance})^\beta$ 
487 struct prob_functor: public thrust::binary_function<float ,float ,
488 float>
489 {
490     const float beta;
491     prob_functor ( float _beta ) : beta ( _beta ) {}
492     __host__ __device__
493     float operator () ( const float & pher , const float & dist
494     ) const
495     {
496         return pher * pow(1 / dist , beta) ;
497     }
498 };
499
500 //randStep: Performs one step of a linear congruential generator
501 struct randStep : public thrust::unary_function<unsigned int ,
502 unsigned int>
503 {

```

```

493  __host__ __device__
494  unsigned int operator()(const unsigned int x) const
495  {
496  //numerical recipies LCG values
497  return ((x * 1664525) + 1013904223) % 4294967296;
498  }
499  };
500
501 //unaryMultiplies: Multiplies all the values of an array by a
    value.
502 struct unaryMultiplies : public thrust::unary_function<int, int>
503 {
504     const int y;
505     unaryMultiplies (int _y) : y (_y) {}
506     __host__ __device__
507     int operator()(const int x) const
508     {
509         return x * y;
510     }
511 };
512
513 //unaryPlus: Adds a value to all the elements of an array.
514 struct unaryPlus : public thrust::unary_function<int, int>
515 {
516     const int y;
517     unaryPlus (int _y) : y (_y) {}
518     __host__ __device__
519     int operator()(const int x) const
520     {
521         return x + y;
522     }
523 };
524
525 //isX: Checks to see if a elements of an array are equal to a
    given constant.
526 struct isX
527 {
528     const int y;
529     isX( int _y ) : y ( _y ) {}
530     __host__ __device__
531     bool operator()(const int x) const
532     {
533         return x == y;
534     }
535 };

```

```

536
537 //Colony: The main ACO functions and data.
538 class Colony
539 {
540 public:
541     Colony(thrust::host_vector<float> newDistances, int
           newNumCities, int newNumAnts);
542     void initialize(); // Initializes data, creates maps and keys,
           performs standard ACO initialization steps etc.
543     void forage(); // Main ACO loop. Performs the solution
           construction step, then updates distances, pheromones,
           probabilities.
544     void computeAntDistances(); // Computes the distances of each
           ant's tour, then updates records.
545     void computeProbabilities(); // Computes the probabilities
           from the distances and pheromones.
546     void setRho(float newRho);
547     void setBeta(float newBeta);
548     double getRho();
549     double getBeta();
550     int getNumAnts();
551     double getIterBestDist();
552     double getGlobBestDist();
553     int getReps();
554     virtual void computeParameters() = 0; //Implemented
           differently in each ACO.
555 protected:
556     float greedyDistance(); // Returns the value of a simple
           greedy solution starting at city 0.
557     virtual void computeInitialPheromone() = 0; //Implemented
           differently in each ACO.
558     virtual void updatePheromones() = 0; //Implemented differently
           in each ACO.
559     //world vars
560     int numCities;
561     int reps;
562     //float alpha = 1, alpha is always 1
563     float beta;
564     float rho;
565     float initialPheromone;
566     thrust::device_vector<float> pheromones;
567     thrust::device_vector<float> distances;
568     thrust::device_vector<float> probabilities;
569     //ant vars
570     int numAnts;

```

```

571 float iterBestDist;
572 float globBestDist;
573 thrust::device_vector<int> iterBestTour;
574 thrust::device_vector<int> globBestTour;
575 thrust::device_vector<float> antVisits;
576 thrust::device_vector<int> toVisit;
577 thrust::device_vector<int> antTours;
578 thrust::device_vector<float> antDistances;
579 thrust::device_vector<float> currentProbabilities;
580 thrust::device_vector<int> currentNeighbors;
581 //maps and keys
582 thrust::device_vector<int> AMapF;
583 thrust::device_vector<int> AMapL;
584 thrust::device_vector<int> tourMap;
585 thrust::device_vector<int> distMap;
586 thrust::device_vector<int> AKey;
587 thrust::device_vector<int> ARepeatCMap;
588 thrust::device_vector<int> ANMapF;
589 thrust::device_vector<int> ANMapL;
590 thrust::device_vector<int> ANKey;
591 thrust::device_vector<int> CKey;
592 thrust::device_vector<int> ARepeatNMap;
593 //scratch variables
594 thrust::device_vector<float> AFloat;
595 thrust::device_vector<int> AInt;
596 thrust::device_vector<int> NInt;
597 thrust::device_vector<int> AInt;
598 thrust::device_vector<int> AInt2;
599 thrust::device_vector<int> AInt3;
600 thrust::device_vector<int> ANInt;
601 thrust::device_vector<float> ACFloat;
602 thrust::device_vector<float> CCFloat;
603 thrust::device_vector<unsigned int> AUnsignedInt;
604 //random numbers
605 thrust::device_vector<unsigned int> ARandom;
606 thrust::device_vector<unsigned int> ACRandom;
607 };
608 #endif
609
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```



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620
621
622 /*****
623 * Colony .cu *
624 * Peter Ahrens *
625 * Main ACO procedures *
626 *****/
627
628 #include "Colony.h"
629
630 //Constructor: Sets defaults and allocates memory.
631 Colony::Colony( thrust::host_vector<float> newDistances , int
newNumCities , int newNumAnts)
632 {
633 //defaults
634 beta = 2;
635 rho = 0.1;
636 //world vars
637 reps = 0;

```

```

638 distances.assign(newDistances.begin(), newDistances.end());
639 numCities = newNumCities;
640 probabilities = thrust::device_vector<float>(numCities*
        numCities);
641 pheromones = thrust::device_vector<float>(numCities*numCities)
        ;
642 //ant vars
643 numAnts = newNumAnts;
644 antDistances = thrust::device_vector<float>(numAnts);
645 antVisits = thrust::device_vector<float>(numCities*numAnts);
646 toVisit = thrust::device_vector<int>(numCities*numAnts);
647 antTours = thrust::device_vector<int>(numCities*numAnts);
648 iterBestDist = std::numeric_limits<float>::max() - 1;
649 globBestDist = std::numeric_limits<float>::max();
650 iterBestTour = thrust::device_vector<int>(numCities);
651 globBestTour = thrust::device_vector<int>(numCities);
652 //maps and keys
653 AMapF = thrust::device_vector<int>(numAnts);
654 AMapL = thrust::device_vector<int>(numAnts);
655 tourMap = thrust::device_vector<int>(numAnts);
656 distMap = thrust::device_vector<int>(numCities*numAnts);
657 AKey = thrust::device_vector<int>(numAnts*numCities);
658 ARepeatCMap = thrust::device_vector<int>(numAnts*numCities);
659 CKey = thrust::device_vector<int>(numCities*numCities);
660 //scratch variables
661 AFloat = thrust::device_vector<float>(numAnts);
662 AInt = thrust::device_vector<int>(numAnts);
663 AInt = thrust::device_vector<int>(numAnts*numCities);
664 AInt2 = thrust::device_vector<int>(numAnts*numCities);
665 AInt3 = thrust::device_vector<int>(numAnts*numCities);
666 AFloat = thrust::device_vector<float>(numAnts*numCities);
667 CFloat = thrust::device_vector<float>(numCities*numCities);
668 AUnsignedInt = thrust::device_vector<unsigned int>(numAnts);
669 //Random numbers
670 ARandom = thrust::device_vector<unsigned int>(numAnts);
671 ACRandom = thrust::device_vector<unsigned int>(numAnts*
        numCities);
672 }
673
674 //initialize: Initializes data, creates maps and keys, performs
        standard ACO initialization steps etc.
675 void Colony::initialize()
676 {
677     //seed the random numbers
678     thrust::transform(thrust::make_counting_iterator(0),

```

```

679     thrust::make_counting_iterator(numAnts*numCities),
680     thrust::make_counting_iterator(time(NULL)),ACRandom.
        begin(),thrust::multiplies<int>());
681 thrust::transform(thrust::make_counting_iterator(0),
682     thrust::make_counting_iterator(numAnts),
683     thrust::make_counting_iterator(time(NULL)),
684     ARandom.begin(),
685     thrust::multiplies<int>());
686 //constant seeds
687 //thrust::transform(thrust::make_counting_iterator(0),thrust::
        make_counting_iterator(numAnts*numCities),thrust::
        make_constant_iterator(1),ACRandom.begin(),thrust::
        multiplies<int>());
688 //thrust::transform(thrust::make_counting_iterator(0),thrust::
        make_counting_iterator(numAnts),thrust::
        make_constant_iterator(1),ARandom.begin(),thrust::
        multiplies<int>());
689 thrust::transform(ACRandom.begin(),
690     ACRandom.end(),
691     ACRandom.begin(),
692     randStep());
693 thrust::transform(ARandom.begin(),
694     ARandom.end(),
695     ARandom.begin(),
696     randStep());
697 //create maps and keys
698 //CCKey
699 thrust::sequence(ACInt.begin(),
700     ACInt.begin()+numCities,
701     0,
702     numCities);
703 thrust::scatter(thrust::make_constant_iterator(1,0),
704     thrust::make_constant_iterator(1,numCities),
705     ACInt.begin(),
706     CCKey.begin());
707 thrust::inclusive_scan(CCKey.begin(),
708     CCKey.end(),
709     CCKey.begin());
710 //ACMapF
711 thrust::sequence(ACMapF.begin(),
712     ACMapF.end(),
713     0,
714     numCities);
715 //ACMapL
716 thrust::transform(ACMapF.begin(),

```

```

717     ACMapF.end() ,
718     thrust::make_constant_iterator(numCities-1),
719     ACMapL.begin() ,
720     thrust::plus<int>());
721 //ACKey
722 thrust::scatter(thrust::make_constant_iterator(1,0) ,
723               thrust::make_constant_iterator(1,numAnts) ,
724               ACMapF.begin() ,
725               ACKey.begin());
726 thrust::inclusive_scan(ACKey.begin() ,
727                       ACKey.end() ,
728                       ACKey.begin());
729 thrust::transform(ACKey.begin() ,
730                 ACKey.end() ,
731                 thrust::make_constant_iterator(-1) ,
732                 ACKey.begin() ,
733                 thrust::plus<int>());
734 //distMap
735 thrust::fill(distMap.begin() ,
736             distMap.end() ,
737             1);
738 thrust::inclusive_scan_by_key(ACKey.begin() ,
739                              ACKey.end() ,
740                              distMap.begin() ,
741                              distMap.begin());
742 thrust::scatter(thrust::make_constant_iterator(0,0) ,
743               thrust::make_constant_iterator(0,numAnts) ,
744               ACMapL.begin() ,
745               distMap.begin());
746 thrust::transform(ACKey.begin() ,
747                 ACKey.end() ,
748                 distMap.begin() ,
749                 distMap.begin() ,
750                 saxpy_functor(numCities));
751 //ARepeatCMap
752 thrust::exclusive_scan_by_key(ACKey.begin() ,
753                              ACKey.end() ,
754                              thrust::make_constant_iterator(1) ,
755                              ARepeatCMap.begin());
756 //ACO Initialize
757 computeParameters();
758 thrust::fill(pheromones.begin() ,
759             pheromones.end() ,
760             initialPheromone);
761 computeProbabilities();

```

```

762 }
763
764 //forage: Main ACO loop. Performs the solution construction
       step, then updates distances, pheromones, probabilities.
765 void Colony::forage()
766 {
767     //initialize variables and select start cities
768     toVisit.assign(ARRepeatCMap.begin(),ARRepeatCMap.end());
769     ACInt2.assign(ACKey.begin(),ACKey.end());
770     thrust::fill(antVisits.begin(),
771                 antVisits.end(),
772                 0);
773     thrust::sequence(tourMap.begin(),
774                    tourMap.end(),
775                    0,
776                    numCities);
777     thrust::transform(ARandom.begin(),
778                     ARandom.end(),
779                     ARandom.begin(),
780                     randStep());
781     thrust::transform(ARandom.begin(),
782                     ARandom.end(),
783                     thrust::make_constant_iterator(numCities),
784                     thrust::make_permutation_iterator(antTours.begin(),
785                                                         tourMap.begin()),
786                     thrust::modulus<unsigned int>());
787     thrust::transform(ACMapF.begin(),
788                     ACMapF.end(),
789                     thrust::make_permutation_iterator(antTours.begin(),
790                                                         tourMap.begin()),
791                     AInt.begin(),
792                     thrust::plus<int>());
793     for(int x = 1; x < numCities; x++)
794     {
795         //update antVisits
796         thrust::scatter(thrust::make_constant_iterator(x,0),
797                        thrust::make_constant_iterator(x,numAnts),
798                        AInt.begin(),
799                        antVisits.begin());
800         thrust::remove_if(thrust::make_zip_iterator(thrust::
            make_tuple(toVisit.begin(),
                       ACInt2.begin()))),
            thrust::make_zip_iterator(thrust::make_tuple(toVisit.begin
                () + ((numCities-x + 1) * numAnts),

```

```

801             ACInt2.begin()+ ((numCities-x + 1) *
802                 numAnts))),
803     antVisits.begin(), isX(x));
804     //get probabilities
805     thrust::transform(thrust::make_permutation_iterator(thrust
806         ::make_permutation_iterator(antTours.begin(), tourMap.
807         begin()), ACInt2.begin()),
808     thrust::make_permutation_iterator(thrust::
809         make_permutation_iterator(antTours.end(), tourMap.end())
810         , ACInt2.begin()+ ((numCities-x) * numAnts)),
811     toVisit.begin(),
812     ACInt.begin(),
813     saxpy_functor(numCities));
814     //update tour map
815     thrust::transform(tourMap.begin(),
816     tourMap.end(),
817     tourMap.begin(),
818     unaryPlus(1));
819     //update random numbers
820     thrust::transform(ACRandom.begin(),
821     ACRandom.begin() + ((numCities-x + 1) * numAnts),
822     ACRandom.begin(),
823     randStep());
824     //select cities
825     thrust::reduce_by_key(ACInt2.begin(),
826     ACInt2.begin()+ ((numCities-x) * numAnts),
827     thrust::make_zip_iterator(thrust::make_tuple(thrust::
828     make_counting_iterator(0),
829     thrust::make_permutation_iterator(
830     probabilities.begin(), ACInt.begin()),
831     ACRandom.begin())),
832     thrust::make_discard_iterator(),
833     thrust::make_zip_iterator(thrust::make_tuple(AInt.
834     begin(),
835     AFloat.begin(),
836     AUnsignedInt.begin()))),
837     thrust::equal_to<int>(),
838     treeSelect());
839     thrust::gather(AInt.begin(),
840     AInt.end(),
841     toVisit.begin(),
842     thrust::make_permutation_iterator(antTours.begin(),
843     tourMap.begin()));
844 }
845 computeAntDistances();

```

```

837     updatePheromones();
838     computeProbabilities();
839 }
840
841 //computeAntDistances: Computes the distances of each ant's tour
842 // , then updates records.
843 void Colony::computeAntDistances()
844 {
845     //compute distances
846     thrust::transform(antTours.begin(),
847                       antTours.end(),
848                       thrust::make_permutation_iterator(antTours.begin(),
849                                                         distMap.begin()),
850                       ACInt.begin(),
851                       saxpy_functor(numCities));
852     thrust::gather(ACInt.begin(),
853                  ACInt.end(),
854                  distances.begin(),
855                  ACFloat.begin());
856     thrust::reduce_by_key(ACKey.begin(),
857                          ACKey.end(),
858                          ACFloat.begin(),
859                          thrust::make_discard_iterator(),
860                          antDistances.begin());
861     //update bests
862     int i = thrust::min_element(antDistances.begin(),
863                                antDistances.end()) - antDistances.begin();
864     thrust::gather(thrust::make_counting_iterator(i*numCities),
865                  thrust::make_counting_iterator((i+1)*numCities),
866                  antTours.begin(),
867                  iterBestTour.begin());
868     iterBestDist = antDistances[i];
869     if(iterBestDist < globBestDist){
870         reps = 0;
871         globBestDist = iterBestDist;
872         globBestTour.assign(iterBestTour.begin(), iterBestTour.end())
873         ;
874     }else{
875         reps++;
876     }
877 }
878
879 //greedyDistance: Returns the value of a simple greedy solution
880 // starting at city 0.
881 float Colony::greedyDistance()

```

```

878 {
879     float distance;
880     int i = 0;
881     int init = i;
882     thrust::device_vector<int> visits(numCities);
883     thrust::fill(visits.begin(),
884                 visits.end(),
885                 1);
886     thrust::device_vector<float> Cfloat(numCities);
887     int j;
888     for(int x = 1; x < numCities; x++){
889         visits[i] = 0;
890         thrust::transform(visits.begin(),
891                          visits.end(),
892                          thrust::make_permutation_iterator(distances.begin(),
893                                                            thrust::make_counting_iterator(i*numCities)),
894                          Cfloat.begin(),
895                          thrust::divides<float>());
896         j = thrust::max_element(Cfloat.begin(),
897                               Cfloat.end()) - Cfloat.begin();
898         distance += distances[numCities * i + j];
899         i = j;
900     }
901     distance += distances[numCities * i + init];
902     return distance;
903 }
904 //computeProbabilities: Computes the probabilities from the
905 //distances and pheromones.
906 void Colony::computeProbabilities()
907 {
908     thrust::transform(pheromones.begin(),
909                     pheromones.end(),
910                     distances.begin(),
911                     probabilities.begin(),
912                     prob_functor(beta));
913 }
914 void Colony::setBeta(float newBeta)
915 {
916     beta = newBeta;
917 }
918 void Colony::setRho(float newRho)
919 {
920

```



```

921     rho = newRho;
922 }
923
924 double Colony::getBeta()
925 {
926     return beta;
927 }
928
929 double Colony::getRho()
930 {
931     return rho;
932 }
933
934 int Colony::getNumAnts()
935 {
936     return numAnts;
937 }
938
939 double Colony::getIterBestDist()
940 {
941     return iterBestDist;
942 }
943
944 double Colony::getGlobBestDist()
945 {
946     return globBestDist;
947 }
948
949 int Colony::getReps()
950 {
951     return reps;
952 }
953
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964
965
966
967 /*****
968 * TSPReader.h *
969 * Dustin Tauxe and Peter Ahrens *
970 * Reads .tsp files *
971 *****/
972
973 #ifndef TSPREADER_H
974 #define TSPREADER_H
975 #include <iostream> // Used for command line I/O
976 #include <fstream> // Used for file Input
977 #include <string>
978 #include <math.h>
979 #include <cctype>
980 #include <float.h> // Used to find maximum float
981 #include <thrust/host_vector.h>
982 using namespace std;
983
984 //TSPReader: Used to read .tsp files.
985 class TSPReader
986 {
987     string name; // TSP name – max length 16
988     int numCities; // Number of cities (dimension)

```

```

989 float* Xcoords; // X coords
990 float* Ycoords; // Y coords
991 string* cityNamees;
992 thrust::host_vector<float> distances; // Distances between
    cities
993
994 public:
995 //Constructors/Destructors
996 TSPReader() {}
997 ~TSPReader();
998
999
1000 bool read(char* file); // Reads a given tsp file and extracts
    data.
1001 string getName();
1002 float* getXcoords();
1003 float* getYcoords();
1004 int getNumNodes();
1005 thrust::host_vector<float> getDistances();
1006 private:
1007 void calculateDistances(); // Calculates distances on the CPU.
1008 };
1009
1010 #endif
1011
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1022
1023
1024 /*****
1025  * TSPReader.cu
1026  * Dustin Tauxe and Peter Ahrens
1027  * Reads .tsp files
1028  *****/
1029
1030 #include "TSPReader.h"
1031
1032 //destructor
1033 TSPReader::~TSPReader()
1034 {
1035     delete [] cityNames;
1036     delete [] Xcoords;
1037     delete [] Ycoords;
1038 }
1039
1040 //read: Reads a given tsp file and extracts data.
1041 bool TSPReader::read(char* fileN)
1042 {
1043     ifstream infile(fileN , ios_base::in);
1044     if(!infile){
1045         cout << "\n" << "Unable to open file: " << fileN << "\n";
1046         return false;
1047     }
1048     string line;
1049     string tag;
1050     string value;
1051     while(infile.good()){
1052         getline(infile ,line);
1053         if(line.length() > 1){

```

```

1054         if(!isprint(line[line.length()-1])) line.erase(line.length
1055             (-1,1);
1056         if(line.find(":") != string::npos){
1057 tag = line.substr(0,line.find(":"));
1058 value = line.substr(line.find(":")+1, line.length()-line.
1059     find(":")-1);
1060     }else{
1061 tag = line;
1062 value = "";
1063     }
1064     while(tag.find(" ") != string::npos){
1065 tag.replace(tag.find(" "),1,"");
1066     }
1067     while(value.find(" ") != string::npos){
1068 value.replace(value.find(" "),1,"");
1069     }
1070     if(tag == "NAME"){
1071 name = value;
1072     }else if(tag == "TYPE"){
1073 if(value != "TSP" && value != "STSP"){
1074     cout << "\n" << "Invalid problem type: " << value << "\n";
1075     return false;
1076 }
1077     }else if(tag == "DIMENSION"){
1078 numCities = atoi(value.c_str());
1079     }else if(tag == "EDGE.WEIGHT.TYPE"){
1080 if(value != "EUC.2D"){
1081     cout << "\n" << "Invalid edge weight type: " << value << "\n
1082     ";
1083     return false;
1084 }
1085     }else if(tag == "NODE.COORD.SECTION"){
1086 //Set coord arrays to appropriate lengths
1087 cityNames = new string [numCities];
1088 Xcoords = new float [numCities];
1089 Ycoords = new float [numCities];
1090 for(int i = 0; infile.good() && i < numCities; i++){
1091     getline(infile, line);
1092     if(!isprint(line[line.length()-1])) line.erase(line.length()
1093         -1,1);
1094     if(line == "EOF"){
1095         return false;
1096     }
1097     cityNames[i] = line.substr(0,line.find(" "));

```

```

1094     Xcoords[i] = atof(line.substr(line.find(" ") + 1, line.
1095         find_last_of(" ") - line.find(" ") - 1).c_str());
1096     Ycoords[i] = atof(line.substr(line.find_last_of(" ") + 1,
1097         line.length() - line.find_last_of(" ") - 1).c_str());
1098 }
1099 }
1100 }
1101 calculateDistances();
1102 return true;
1103 }
1104
1105 //calculateDistances: Calculates distances on the CPU.
1106 void TSPReader::calculateDistances() {
1107     distances = thrust::host_vector<float> (numCities*numCities);
1108     float k;
1109     for(int i = 0; i < numCities; i++){
1110         for(int j = 0; j < numCities; j++){
1111             k = sqrt(pow(Xcoords[i]-Xcoords[j],2)+pow(Ycoords[i]-
1112                 Ycoords[j],2));
1113             if(i == j)
1114                 k = std::numeric_limits<float>::max();
1115             distances[i * numCities + j] = k;
1116         }
1117     }
1118
1119 string TSPReader::getName()
1120 {
1121     return name;
1122 }
1123
1124 float* TSPReader::getXcoords()
1125 {
1126     return Xcoords;
1127 }
1128
1129 float* TSPReader::getYcoords()
1130 {
1131     return Ycoords;
1132 }
1133
1134 int TSPReader::getNumNodes()
1135 {

```

```

1136     return numCities;
1137 }
1138
1139 thrust::host_vector<float> TSPReader::getDistances()
1140 {
1141     return distances;
1142 }
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1178
1179
1180 /*****
1181 * Comm.h
1182 * Peter Ahrens
1183 *****/

```

```

1159 * Communicates over a pipe *
1160 *****/
1161
1162 #ifndef COMMH
1163 #define COMMH
1164 #include <iostream>
1165 #include <sstream>
1166 #include <iomanip>
1167 #include <unistd.h>
1168 #include <stdlib.h>
1169 #include <string>
1170 #include <cctype>
1171 using namespace std;
1172
1173 //Comm: A class used to simplify the data transfer over a pipe.
1174 class Comm
1175 {
1176 public:
1177     Comm(int read, int write); //constructor: Takes as arguments
1178         the necessary pipes to work with.
1179     ~Comm();
1180     string recieve(); //recieve: Looks for data on the pipe. If
1181         there is some, it is returned. If not, an empty string is
1182         returned.
1183     bool send(string message); //send: Puts the given data on the
1184         pipe
1185 private:
1186     int tagLength;
1187     int readPipe;
1188     int writePipe;
1189     string intToString(int t, int padding); //intToString:
1190         Converts an int to a specified size string with 0s as
1191         padding.
1192 };
1193 #endif
1194
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1199
1200
1201 /*****
1202 * Comm.cpp *
1203 * Peter Ahrens *
1204 * Communicates over a pipe *
1205 *****/
1206
1207 #include "Comm.h"
1208
1209 //constructor: Takes as arguments the necessary pipes to work
with.
1210 Comm::Comm(int newReadPipe, int newWritePipe)
1211 {
1212     readPipe = newReadPipe;
1213     writePipe = newWritePipe;
1214     tagLength = 8;
1215 }
1216
1217 Comm::~Comm() //Destructor.
1218 {}
1219

```

```

1220 //recieve: Looks for data on the pipe. If there is some, it is
      returned. If not, an empty string is returned.
1221 string Comm::recieve()
1222 {
1223     char tag[tagLength];
1224     int rv = read(readPipe,tag,tagLength);
1225     if(rv < 0){
1226         cout << "Read Error 1";
1227         exit(1);
1228     }
1229     if(rv == 0){
1230         return string("");
1231     }
1232     string inputTag = tag;
1233     int toRead = atoi(inputTag.substr(0,tagLength).c_str());
1234     char charIn[toRead];
1235     if(read(readPipe,charIn,toRead) < 0){
1236         cout << "Read Error 2";
1237         exit(1);
1238     }
1239     string output = charIn;
1240     output = output.substr(0,toRead);
1241     return output;
1242 }
1243
1244 //send: Puts the given data on the pipe
1245 bool Comm::send(string message)
1246 {
1247     message = intToString(message.length(),tagLength) + message;
1248     return(write(writePipe,message.data(),message.length()) > 0);
1249 }
1250
1251 //intToString: Converts an int to a specified size string with 0
      s as padding.
1252 string Comm::intToString(int t, int padding)
1253 {
1254     std::ostringstream oss;
1255     oss << setfill('0') << setw(padding) << t;
1256     return oss.str();
1257 }
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```

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1269
1270
1271 /*****
1272  * Writer.h
1273  * Dustin Tauxe and Peter Ahrens
1274  * Writes output to file and stdout
1275  *****/
1276
1277 #ifndef WRITER_H
1278 #define WRITER_H
1279 #include <iostream>
1280 #include <iomanip>
1281 #include <fstream>
1282 #include <cstdlib>
1283 #include <time.h>
1284 using namespace std;

```

```

1285
1286 class Writer
1287 {
1288     public:
1289         Writer(); // Sets defaults.
1290         Writer(char* file); // Sets defaults and opens the given file
1291
1292         ~Writer();
1293         bool setFile(char* file); // Tries to open given file. If it
1294             does, it is changed to writing mode.
1295         void writeHeader(float beta, float rho, int numAnts, string ACO
1296             , string TSPName); // Writes a header to the file and (if
1297             in writing mode) to the file.
1298         void write(int iter, double iterBest, double globBest, double
1299             time, double iterTime); // Writes a standard line of output
1300             to stdout and (if in writing mode) to the file.
1301     private:
1302         char* fileName;
1303         ofstream f; // this is the file
1304         char* temp; // scratch
1305         bool writing;
1306 };
1307 #endif
1308
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1314
1315
1316
1317 /*****
1318  * Writer.cpp
1319  * Dustin Tauxe and Peter Ahrens
1320  * Writes output to file and stdout
1321  *****/
1322
1323 #include "Writer.h"
1324
1325 //Constructor: Sets defaults.
1326 Writer::Writer()
1327 {
1328     writing = false;
1329 }
1330
1331 //Constructor: Sets defaults and opens the given file.
1332 Writer::Writer(char* file)
1333 {
1334     setFile(file);
1335 }
1336
1337 Writer::~~Writer()//Destructor.
1338 {
1339     delete [] fileName;
1340 }
1341
1342 //setFile: Tries to open given file. If it does, it is changed
      to writing mode.
1343 bool Writer::setFile(char* file)
1344 {

```

```

1345 fileName = file;
1346 writing = true;
1347 f.open(file, ios_base::app);
1348 if (!f){
1349     writing = false;
1350 }
1351 return writing;
1352 }
1353
1354 //writeHeader: Writes a header to the file and (if in writing
1355 //mode) to the file.
1356 void Writer::writeHeader(float beta, float rho, int numAnts,
1357 string ACO, string TSPName)
1358 {
1359     time_t rawtime;
1360     time (&rawtime);
1361     if(writing){
1362         f << "\n" << "Date: " << ctime (&rawtime) <<
1363         "TSP: " << TSPName << "\n" <<
1364         "ACO: " << ACO << "\n" <<
1365         "numAnts: " << numAnts << "\n" <<
1366         "Alpha: 1 " << "Beta: " << beta << " Rho: " << rho << "\n"
1367         <<
1368         "Iteration, Iteration_Best, Global_Best, Time,
1369         Iteration_Time\n" << flush;
1370     }
1371     cout << "\n" << "Date: " << ctime (&rawtime) <<
1372     "TSP: " << TSPName << "\n" <<
1373     "ACO: " << ACO << "\n" <<
1374     "numAnts: " << numAnts << "\n" <<
1375     "Alpha: 1 " << "Beta: " << beta << " Rho: " << rho << "\n"
1376     <<
1377     std::left << setw(10) << "Iteration" << setw(10) << "
1378     Iter_Best" << setw(10) << "Glob_Best" << setw(10) << "
1379     Time" << setw(10) << "Iter_Time" << "\n";
1380 }
1381
1382 //write: Writes a standard line of output to stdout and (if in
1383 //writing mode) to the file.
1384 void Writer::write(int iter, double iterBest, double globBest,
1385 double time, double iterTime)
1386 {
1387     if(writing){
1388         f << iter << "," << iterBest << "," << globBest << "," <<
1389         time << "," << iterTime << "\n" << flush;

```

```

1380 }
1381 cout << std::left << setw(10) << iter << setw(10) << iterBest
      << setw(10) << globBest << setw(10) << time << setw(10) <<
      iterTime << "\n";
1382 }
1383
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```

11 Acknowledgements

Thanks so much to our mentors and to Mr. Goodwin for supporting and guiding us along the past couple years of computing. We have come a long way.

References

- [1] Peter's Computer: Asus G53J, Intel i7-740QM 1.73GHz, 8GB Memory, Nvidia GeForce GTX 460M ; VRAM: 1.5GB, running Ubuntu Linux 11.10.
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